High-resolution Doppler-velocity estimation techniques for processing of coherent heterodyne pulsed lidar data

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On the basis of an analysis of the autocovariance of the complex heterodyne signal, some novel algorithms are derived and are investigated for use in determining, with high spatial resolution, Doppler-velocity coherent-lidar profiles in the case of rectangular and rectangular-like sensing laser pulses. These algorithms generalize other known Doppler-velocity estimators for the more complex case of nonuniform scattering and Doppler-velocity distribution within the pulse length. Algorithm performance and efficiency are studied and are illustrated by computer simulations. It is shown that the Doppler-velocity profiles can be determined with essentially better resolution in comparison with the use of other known estimation approaches, but at the expense of some increase in the number of statistical realizations (number of laser shots) required to reduce the speckle-noise effect. The minimum achievable resolution interval is shown to be much shorter than the pulse length.

1. INTRODUCTION

The range (spatial) resolution $R$ of pulsed coherent Doppler lidars is usually accepted to be of the order of the pulse length. At the same time, the range resolution and the Doppler-velocity resolution (sensitivity) $\Delta v$ are reciprocally related, and their product is proportional to the wavelength $\lambda$ of the laser radiation. That is, $R \Delta v = c \lambda / 4$, where $c$ is the speed of light. Thus, at a given laser radiation wavelength, the pulse length should exceed some minimum value determined by the minimum Doppler-velocity change to be resolved. So the use of CO$_2$ laser radiation ($\lambda = 10.6 \mu$m) has numerous advantages and requires a minimum pulse duration of the order of microseconds ($\sim 5.3 \mu$s) for achieving a Doppler-velocity resolution of 1 m/s. Then the corresponding spatial resolution cell along the line of sight is several hundred meters ($\sim 800$ m) long. Within such a long-resolution cell, the dispersion of the velocity and concentration of atmospheric particulate matter might be quite large, but the information about it is lost. The use of shorter pulses of shorter-wavelength radiation is a way to improve the range resolution without lowering the velocity resolution, as in the case of employing laser pulses with $\sim 1-\mu$s duration (150-m range resolution) and $\lambda \sim 2 \mu$m.5,7

Another approach for improving the range resolution is to develop inverse mathematical techniques for retrieving Doppler-velocity profiles with a considerably shorter resolution cell in comparison with the pulse length. Recently we developed a similar approach concerning the lidar signal power profile. It is based on deconvolution techniques for retrieving the so-called short-pulse ($\delta$-pulse) power profile on the basis of the measured power profile and the known pulse shape. In the present study we take a further step: That is, on the basis of an analysis of the autocovariance of the complex heterodyne signal, we develop some novel inverse techniques for improving the coherent-lidar spatial resolution with respect to the Doppler-velocity profile in the case of rectangular and rectangular-like sensing laser pulses. The inverse algorithms obtained generalize other known Doppler-velocity estimators for the more complex case of nonuniform scattering and Doppler-velocity distribution within the pulse length. Although the rectangular pulse shape is an idealization, it may be a good approximation of some real laser pulses. Moreover, many features of the techniques developed here, whose feasibility is also tested and illustrated below by computer simulations, should be inherent in similar techniques concerning the cases of more general (or even arbitrary) pulse shapes.

2. COHERENT LIDAR RETURN SIGNAL

The approach considered in this section for the description and modeling of coherent monostatic lidar signals was to our knowledge, first used in Refs. 13 and 14 and was substantiated in Ref. 15. It is described here from a somewhat different point of view in order to make clear the following analysis.

We shall consider the sensing radiation as a sequence of quasi-monochromatic laser pulses with basic (nominal) frequency $\omega_0$, dimensionless temporal amplitude envelope $f_\delta(\vartheta)$, and some regular frequency deviation (chirp) $\delta\omega_0(\vartheta)$ ($|\delta\omega_0| \ll \omega_0$); $\vartheta$ is a time variable, and $f_\delta(\vartheta) = 0$ for $\vartheta < 0$. Then the expression describing the space-
time amplitude and phase distribution of the sensing laser radiation field $E_0(r, t)$ may be written in the form

$$E_0(r, t) = \mathcal{P}A(r) f(t - z/c) \exp \left[ j \omega_0 (t - z/c) + \int_0^{t-z/c} \delta \omega_0(t') dt' + \phi_0(t - z/c) \right], \quad (1)$$

where $\phi_0(t)$ is a random function describing temporal phase fluctuations, $j$ is imaginary unity, $c$ is the speed of light, $t$ is the time interval after the pulse emission, $\mathcal{P}$ is an in general complex vector characterizing the field polarization state, and $A(r)$ is a space-dependent factor in which, except for the aperture diffraction, the atmospheric influence (extinction, turbulence) along the line of sight is also taken into account (see, e.g., Ref. 16). The line of sight coincides with the axis of the pulsed laser beam. The vector coordinate $r = \{ \rho, z \}$ consists of a transverse vector coordinate $\rho$ and a longitudinal coordinate $z$ with respect to the line of sight, $\rho = 0$ on the beam axis, and $z = 0$ at the transceiver aperture plane.

In the case of linear polarization of the incident laser radiation, $\mathcal{P} = \mathbf{e}$ is a real unit vector. Then the field vector $E_0(t, t')$ of the radiation backscattered by one (say, $i$th) aerosol particle, at the transceiver aperture plane $\{ \rho_i, z \}$ and moment $t$ after the pulse emission, is expressible in the form

$$E_{0i}(t, t') = A_i(r_i(t')) f_i[t - 2z_i(t')/c] \exp \left[ j \omega_0 (t - 2z_i(t')/c) + \int_0^{t-2z_i(t')/c} \delta \omega_0 (t') dt' + \phi_0 (t - 2z_i(t')/c) \right] \times a_i(r_i(t')) G_i(\rho_i ; r_i(t')), \quad (2)$$

where $r_i(t') = \{ \rho_i(t'), z_i(t') \}$ is the vector coordinate of the scattering particle at the corresponding earlier moment of interaction $t'$, $a_i$ is the scattering amplitude of the particle, and $G_i(\rho_i ; r_i)$ is the paraxial approximated Green function used for description of the backscattered-wave propagation in the atmosphere, taking into account in general the extinction and turbulence effects.

Under single-scattering conditions, the total backscattering field $E_i(\rho_i, t)$ is a superposition of the elementary wave fields backscattered by the scatterers that contribute to the signal. That is,

$$E_i(\rho_i, t) = \sum_i E_{0i}(\rho_i, r_i(t'), t). \quad (3)$$

The equivalent local-oscillator field vector $E_{\phi}(\rho_i, t)$ at the transceiver aperture plane can be given in the form

$$E_{\phi}(\rho_i, t) = A_{\phi}(\rho_i) \exp [j \omega_{\phi}(t) + \phi_\phi(t)], \quad (4)$$

where $A_{\phi}(\rho_i)$ is the spatially dependent (complex amplitude) factor, $\omega_{\phi}$ is the optical heterodyne frequency, and $\phi_\phi(t)$ is a function describing phase fluctuations.

When the photodetector collects all the local-oscillator beam energy and all the backscattered radiation that is covered by the receiving optical system, the complex photocurrent $I(t)$ resulting from coherent heterodyne detection may be represented as

$$I(t) = J(t) + jQ(t) = K \int E_{\phi}(\rho_i, t) \cdot E_{0i}(\rho_i, t) d\rho_i, \quad (5)$$

where $K = 2\varepsilon q / \hbar \omega$, $q$ is the photodetector (uniform) quantum efficiency, $\varepsilon$ is the electron charge, $\hbar = h/2\pi$, $h$ is the Planck constant, $\omega = (\omega_o + \omega_i)/2$, and superscript $c$ denotes complex conjugation. Functions $J(t)$ and $Q(t)$ are, respectively, in-phase and quadrature components of $I(t)$. On the basis of Eqs. (2)–(5) we obtain the following explicit expression of $I(t)$:

$$I(t) = K \exp [j \omega_o - \omega_i] t - \phi_\phi(t)] \sum_i a_i A_i(r_i(t')) \times f_i[t - 2z_i(t')/c] \exp \left[ j \omega_0 (t - 2z_i(t')/c) + \int_0^{t-z_i/t'} \delta \omega_0 (t') dt' + \phi_0 (t - 2z_i(t')/c) \right] \times a_i(\rho_i) G_i[\rho_i ; r_i(t')]) d\rho_i, \quad (6)$$

One can assume that when interacting with a propagating (sensing) laser pulse, each ($i$th) scatterer moves with a constant radial velocity $v_i < c$. Then one can derive the Doppler frequency shift on the basis of classical considerations by representing $z_i(t')$ in the exponent in Eq. (6) as $z_i(t') = z_{io} + v_i(t - 2z_{io}/c)$, where $z_{io}$ is the longitudinal position of the particle when it is met by the pulse front. The corresponding total vector coordinate is $r_{io} = \{ \rho_{io}, z_{io} \}$. Factors $A_i$ and $G_i$ in Eq. (6) also depend, in general, on the particle motion as $r_i(t') = r_{io} + v_i(t - 2z_{io}/c)$. Here $v_i = [v_{i1}, v_{i2}, v_{i3}]$ is the total vector of the scatterer velocity with transverse component $v_{i2}$. According to the results of Ref. 19 (see also Ref. 20), the indicated dependence does not influence the temporal autocovariance of $I(t)$ which is of interest in what follows) when the inequalities $v_T T / a < 1$ and $\tau a c / \tau a c (\lambda_{c}) < 1$ are satisfied; $v_T = [v_{T1}, v_{T2}, v_{T3}]$, $\tau$ is the pulse duration, and $a$ is the laser beam radius. Since these inequalities are usually satisfied for coherent lidars, we shall neglect the effect of the scatterer motion on $A$ and $G_i$. Thus Eq. (6) may be rewritten in the form

$$I(t) = K \exp [j \omega_0(t) + \phi_\phi(t)] \sum_i a_i A_i(r_i)f_i(t - 2z_{io}/c) \times \exp [-2i\hbar z_{io} / c] \times \exp [j \omega_0(t - 2z_{io}/c) + \phi_\phi(t - 2z_{io}/c)] \times a_i(\rho_i) G_i[\rho_i ; r_i(t')]) d\rho_i, \quad (7)$$

where $\lambda_{c}$ is the wavelength of the laser beam.
where $\omega_{m1} = \omega_p - \omega_c$ is the intermediate frequency, 
$\omega_{m2} = \omega_c/\chi_1$ is the Doppler-shifted frequency and $k_D = \omega_m/c$ is the corresponding wave number, $\chi_1 = 1 - 2v/c$, and 
$$
\phi_{dM}(t - 2z/c) = \frac{1}{\lambda} \int_0^t \phi_0(\tau) d\tau
$$
is the phase increment due to the chirp. Consider the semispace (z > 0) as consisting of plane slices with thickness $\Delta z_o = c\Delta t/2$ that are perpendicular to the line of sight. The slice thickness $\Delta z_o$ may in practice be equal to the spatial sampling interval along the line of sight that corresponds to a temporal sampling interval $\Delta t_o$. We shall assume that $\Delta z_o$ is large compared with the wavelength $\lambda = 2\pi/c\omega_c$ but that it is sufficiently small that within each (lth) slice $\Delta t_o < z < \Delta z_o$ functions $\phi_0$, $\phi_r$, $\phi_c$, $A$, $G_1$, and certainly $f$ do not change with $z$, and the scattering particles have uniform radial velocity $v(z)$. The scatterers contributing to the signal may be numbered within every lth slice by an additional (except l) index n $(l, n = 1, 2, \ldots)$. So instead of Eq. (7) we can write

$$
I(t) = K \exp(-j\phi_h(t)) \sum_n f_n(t - 2z_l/c) \times \left\{ e^{j(\omega_m t + \phi_{dM}(t - 2z_l/c) \chi_l)} + \phi_0(t - 2z_l/c) \chi_l \right\} dA(z_l),
$$
where 
$$
z_l = \Delta z_o, \quad \omega_{m1} = \omega_c/\chi_1, \quad \chi_l = 1 - 2v/c, (8)
$$
and 
$$
dA(z_l) = \sum_{l=0}^\infty \exp(2j.k_{Dz}z_l/\lambda) A_p(n, z_l) a_{nl}
$$

(9)
are coordinates concerning the nth particle inside the lth slice, $a_{nl}$ is the corresponding scattering amplitude, and $k_D = \omega_m/c$. The aerosol scatterers have a quite random mutual disposition that conditions the inherent character of the scattering process. In this case, when the single backscattering prevails $dA(z_l) = dA(z_l) + j\Delta A(z_l)$ is a circular complex Gaussian random quantity. The distributions of $dA(z_l)$ and $\Delta A(z_l)$ are identical Gaussian ones with zero mean value and variance $D[A(z_l)] = \langle|dA(z_l)|^2\rangle = \langle|\Delta A(z_l)|^2\rangle$, where $\langle \cdot \rangle$ denotes ensemble average. Also, $dA(z_l)$ and $\Delta A(z_l)$ are statistically uncorrelated i.e., $\langle dA(z_l)\Delta A(z_l) \rangle = 0$. Because of the incoherent scattering, the contributions of different slices to the heterodyne signal are also uncorrelated, and correspondingly $\langle \phi(z)\Delta A(z_l) \rangle = 0$ when $s \neq 0$ ($s = \pm 1, \pm 2, \ldots$). The value of $dA(z_l)$ can be represented in the form

$$
dA(z_l) = \Phi_o^{1/2}(z_l)(\Delta z_o)^{1/2} w_l, (10)$$
where

$$
\Phi_o(z_l)(\Delta z_o) = \langle|dA(z_l)|^2\rangle = D[A(z_l)] + D[\Delta A(z_l)]
$$
is the variance of $dA(z_l)$, and $w_l = w = w_r + jw_i$ is a complex random variable with zero mean value $\langle w \rangle = \langle w_r \rangle = \langle w_i \rangle = 0$ and unit variance $Dw = \langle w^2 \rangle = \langle w_r^2 \rangle + \langle w_i^2 \rangle = 1$. $Dw_r = \langle w_r^2 \rangle = Dw_i = \langle w_i^2 \rangle$, $\langle w, w_r \rangle = 0$, and $\langle w, w_i \rangle = 0$ for $s \neq 0$. Taking into account the properties of $dA(z_l)$, the mean heterodyne signal power $P(t) = \langle |I(t)|^2 \rangle$ is obtainable from Eq. (8) in the form of the following sum,

$$
P(t) = \sum_{l=1}^l f(t - 2z_l/c) \Phi(z_l) \Delta z_o (11a)$$
or integral

$$
P(t) = \int f(t - 2z/c) \Phi(z) dz, (11b)$$
where $f = f^2$ and $\Phi(z) = K^2 \Phi_o(z)$. Thus the quantity $\Phi(z)$ characterizes the contribution to the heterodyne signal power of unitary length along the line of sight at unit $f$. The function $f(\delta) = P_{imp}(\delta)/P_P$ describes the pulse power shape $P_{imp}(\delta) = |E_p(\rho, \delta) \cdot E^*(\rho, \delta) d\rho |^2$ normalized to its peak value $P_p = \int |E_p(\rho)A_p(\rho) d\rho|^2$ [see Eq. (1) with $\mathcal{F} = \rho_s, \mathcal{P} = E$]. Correspondingly, the pulse energy $E_{imp} = \int P_{imp}(\delta) d\delta = P_p \tau$, where $\tau = \int f(\delta) d\delta$ is the effective pulse duration $(\tau_c = \tau)$ is then the effective pulse length. According to Eqs. (11a) and (11b), the value of $\Phi(z)$ (or $\Phi(z_l)$) is proportional to the maximum-resolved (short-pulse) signal power profile $P_m(t - 2z/c) = C_n \Phi(z) d\delta$ that is obtainable by sufficiently short-sensing pulses (physical $\delta$ pulses). In the case of a restricted pulse shape with actual duration $\tau_n$ when $f(\delta) = \delta$ in Eqs. (11a) and (11b) we have $I_1 = c(t - \tau_n)/(2\Delta z_o), I_2 = c(t + \tau_n)/(2\Delta z_o)$, and $\tau(t) = c(t - \tau_n)/2$. In the case of pulse shape with asymptotically falling tail, $I_1 = z_o/\Delta z_o, I_2 = c(t/2\Delta z_o)$, and $\tau(t) = z_o$. The range from $z = 0$ to $z = z_o$ is the lidar dead zone. The backscattered radiation from this zone is not detectable because, for example, there is no overlapping between the sensing laser beam and the field of view of the receiving optical system.25

Taking into account Eq. (10), we can rewrite Eq. (8) in the following forms:

$$
I(t) = \exp[-j\phi_h(t)] \sum_{l=1}^l \left[ f(t - 2z_l/c) \Phi(z_l) \Delta z_o \right]^{1/2} \times w_l(z_l) \exp[j(\omega_{m1} t + \phi_{dM}(t - 2z_l/c) \chi_l)] + \phi_0(t - 2z_l/c) \chi_l), (12a)$$
or

$$
I(t) = \exp[-j\phi_h(t)] \int \phi_{dM}(t - 2z/c) \chi(z) dz) + \phi_0(t - 2z/c) \chi(z)] dA(z), (12b)$$
where

$$
\langle dA(z) dA(z') \rangle = \Phi(z) \delta(z - z') dz dz',
$$
\omega_m(z) = \omega_0(z) - \omega_h, \chi(z) = 1 - 2v/c,$
and $\phi_0$ is assumed to be stable within a slice.

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3. AUTOCOVARIANCE OF THE COHERENT LIDAR RETURN SIGNAL

On the basis of Eq. (12a) or Eq. (12b), assuming that the value of \( \phi_0 \) is stable within a pulse duration, we obtain the following expression of the autocovariance function of the coherent lidar signal:

\[
\text{Cov}(t, \theta) = (I^a(t)I(t + \theta)) = \int_{t \pm t/2} d\phi z f_0(t - 2z/c)f_0(t + \theta - 2z/c)\Phi(z) \\
\times \exp[j(\omega_m(t + \phi) + \Delta\phi_{0}(t, \theta, z))] \\
\times \xi(t, \theta, z) \gamma_0\left(2\omega_m/\theta c\right),
\]

(13a)

where \( \theta > 0 \) is a time shift,

\[
\omega_m(z = ct/2) = \theta^{-1} \times \arctan\left[\frac{\text{Im} \text{Cov}'(t, \theta) + \Phi(c(t + \theta - t/2))\gamma \sin(\omega_m(c(t + \theta - t/2)\theta))}{\text{Re} \text{Cov}'(t, \theta) + \Phi(c(t + \theta - t/2))\gamma \cos(\omega_m(c(t + \theta - t/2)\theta))}\right],
\]

(14a)

where \( \gamma = \gamma(z = c(t + \theta - t)/2; 2\omega_m/\theta c) \) and the quantities \( ct/2, [c(t + \theta - t)/2] \), and \( c(t + \theta - t)/2 \) are arguments of the functions \( \Phi \) and \( \omega_m \). It can be seen that the second relation obtained [formula (14b)] depends explicitly on the velocity-fluctuation characteristic function \( \rho(z, 2\omega_m/\theta c) = \gamma(z, 2\omega_m/\theta c) \), which is in addition assumed here [only when deriving Eq. (14b)] to be a real function corresponding to some symmetric probability density distribution \( p(\nu) = p(-\nu) \). The last assumption is in accordance, for example, with the concept of locally homogeneous and isotropic (statistically) turbulent velocity fluctuations (see, e.g., Ref. 23). Thus it turns out that formula (14b) is effective only when \( \omega_m\theta/c \ll 1 \) or \( z_n < c(t - \theta)/2, \) where \( \sigma = (\bar{\bar{v}}^r)^{1/2} \) is the root-mean-square value of the velocity fluctuations. In the latter case

\[
\omega_m(z = ct/2) = \theta^{-1} \arctan[\text{Im} \text{Cov}'(t, \theta)/\text{Re} \text{Cov}'(t, \theta)],
\]

(14c)

which means that at sufficiently long sensing distances \( \nu \) and lidar dead zone \([0, z_n]\), one could determine \( \omega_m \) at long distances \( z \) without any prior (a priori) information about \( \gamma(z, 2\omega_m/\theta c) \).

In the cases when \( \Phi(z) \) and \( \omega_m(z) \) are practically constant within every \( c\tau/2 \)-long spatial interval, Eq. (13b) leads to the algorithms

\[
\omega_m(z = ct/2) = [\text{Im} \text{Cov}'(t, \theta = 0)/\text{Cov}(t, \theta = 0)],
\]

(15a)

\[
\omega_m(z = ct/2) = \theta^{-1} \arctan[\text{Im} \text{Cov}(t, \theta)/\text{Re} \text{Cov}(t, \theta)],
\]

(15b)

where the first one has been investigated by us formerly and the second one is the well-known pulse-pair (PP) algorithm.11 Thus, formulas (14a)–(14c) generalize the known algorithms (15a) and (15b) for the more complex case of nonuniform distributions of \( \Phi(z) \) and \( \omega_m(z) \) within the pulse length.

The resolution step achievable by algorithms (14a)–(14c) may be considerably shorter than \( c\tau/2 \) and even of
the order of the sampling interval. In this case Cov\( (t, \theta) \) is determined by a “true” ensemble averaging over a sufficiently large number of laser shots to suppress the noise effects (see below in Section 5). In this way the signal power profile \( P(t) \) is also determined because \( P(t) = \text{Cov}(t, \theta) = 0 \). The short-pulse signal power profile \( \Phi(z = ct/2) \) is then obtained (by the deconvolution technique) as a solution of Eq. (13c). The PP algorithm makes use in practice of a temporal averaging (along the line of sight) on the basis of one realization (or relatively small number of realizations) of the random process \( I(t) \) assumed to be the locally stationary and ergodic one. This allows one to achieve a fast result but lowers the spatial resolution because the resolution cell becomes longer than \( c\tau/2 \).

5. SIMULATIONS

The algorithm performance has been simulated by use of various distributions of the radial velocity \( v(z) \) and the mean signal power \( \propto \Phi(z) \). Below we present results for two models of \( v(z) \). The first model is (see also Fig. 1)

\[
v(z) = v(z - z_o) = v_1(z)\sin(4\pi(z - z_o)/[q_1\lambda + q_2(z - z_o)]) + v_o,
\]

where

\[
v_1(z) = q_3(z - z_o)/(z_o - z_o) + q_4, \quad q_1 = 1.5 \times 10^8, \quad q_2 = 0.3, \quad q_3 = 10 \text{ m/s},
\]

\[
q_4 = 3 \text{ m/s}, \quad v_o = 3 \text{ m/s}, \quad z_o = 8700 \text{ m}.
\]

This model is like an alternating wind velocity profile and is intended for investigating the algorithm performance over relatively large ranges along the line of sight. The corresponding model used of \( \Phi(z) \) is (see Fig. 2)

\[
\Phi(z = ct/2) = W \begin{cases} 
0 & \text{for } t \leq t_o \\
B_1(t - t_o)^{-3} \exp[-B_2/(t - t_o)] + B_3 \sin^2[2\pi(t - t_o)/T] & \text{for } t_o < t \leq B_2 + t_o, \\
B_1(t - t_o)^{-3} \exp[-B_2/(t - t_o)] & \text{for } t > B_2 + t_o.
\end{cases}
\]

where \( W \) is a system constant, \( B_1 = 3000 \mu s^3 \), \( B_2 = 20 \mu s \), \( B_3 = 0.1 \), and \( T = 10 \mu s \). At a laser radiation wavelength assumed as \( \lambda = 10.6 \mu m \), the temporal sampling interval \( \Delta t_o = 2\Delta z_o/c \), the pulse duration \( \tau = l_p/c \), and the initial moment of signal registration \( t_o = 2\Delta z_o/c \) may have various values; \( \Delta z_o \) and \( l_p \) are the spatial sampling interval and the pulse length, respectively. The concrete values concerning the models shown in Figs. 1 and 2 are \( \Delta t_o = 0.1 \mu s \) (\( \Delta z_o = 15 \text{ m} \)), \( \tau = 4 \mu s \) (\( l_p = 1200 \text{ m} \)), and \( t_o = 2 \mu s \) (\( z_o = 300 \text{ m} \)).

The second model of \( v(z) \) is (see Fig. 3)

\[
v(z) = C[(z - z_o) - a] \exp[-((z - z_o) - a)^2/b^2]/b^2,
\]

where \( a = 97.5 \text{ m}, b = 22.5 \text{ m}, \) and \( C = -1050 \mu m^2/\mu s \). It describes a wind-vortex-like distribution of the radial velocity \( v(z) \). Such a distribution is characterized by small spatial size and sharp velocity variations. Therefore it is convenient for investigating (by simulations) the high-resolution performance potential of algorithms (14a) and (14b). The temporal sampling interval here is chosen to be \( \Delta t_o = 0.01 \mu s \) (\( \Delta z_o = 1.5 \text{ m} \)). The pulse durations used in the simulations are \( \tau = 200 \text{ ns} \) (\( l_p = 60 \text{ m} \)) at \( \lambda = 2 \mu m \) and \( \tau = 4 \mu s \) (\( l_p = 1200 \text{ m} \)) at \( \lambda = 10.6 \mu m \). As above, the initial moment \( t_o \) of signal
registration may have various values. The model used here of \( \Phi(z) \) is given again by Eq. (17) but with a new set of constants \( (B_1 = 20 \, \mu s^3, B_2 = 3.5 \, \mu s, B_3 = 0.05, \) and \( T = 1 \, \mu s) \). The coherent-lidar return signal \( I(t) = J(t) + |Q(t) |^2 \) is simulated according to Eqs. (12a) and (12b) (with \( \phi_h = \phi_v = \phi_o = 0, \bar{v} = 0, \) and \( \omega_h = \omega_v \)), where the complex random variable \( w = w_r + jw_i \) (i.e., \( w_r \) and \( w_i \)) should be generated in an appropriate way. We have solved this task by using the method described in Ref. 24. As a test, we have compared the probability density distribution of the random quantity \( |I(t)|^2/P(t) \) (the signal power normalized to its mean value) with the corresponding expected negative exponential distribution.\(^{21}\) As is seen in Fig. 4, the coincidence is satisfactory. A pair of realizations of the in-phase \( J(t) \) and the quadrature \( Q(t) \) components of the signal are given, for example, in Fig. 5, for the case of the first Doppler-velocity model described above [Eq. (16)]. The covariance estimates \( \hat{\text{Cov}}(t, \theta) \) are obtained on the basis of the relation

\[
\hat{\text{Cov}}(t, \theta) = \sum_{k=1}^{N} I_k^*(t_{k2})I_k(t_{k2} + m \Delta t_o),
\]

where \( N \) is the number of realizations employed, \( I_k(t) \). The same relation, at \( m = 0 \), provides the signal power profile estimates \( \hat{P}(t) = \hat{\text{Cov}}(t, \theta = 0) \). Then the estimates \( \hat{\Phi}(z = ct/2) \) of the short-pulse signal power profile \( \Phi(z = ct/2) \) are obtained (by deconvolution\(^{21}\)) as the solution of Eq. (13c) with \( P(t) = \hat{P}(t) \). This solution has the form \( \hat{\Phi}(z = ct/2) = \hat{\Phi}_\omega(z = c(t - \tau)/2) + (2c)\int \hat{P}(t), \hat{\Phi}(z = \tau) = 0 \). The simulations performed show that, for instance, an additive noise at signal-to-noise ratio \( S/N \) (defined as, e.g., in Ref. 25) of the order of 100, at maximum signal power, does not have a noticeable effect on the final results when \( N = 500 \). Therefore, we have investigated mainly the influence of speckle noise on algorithm performance. A reduction of the influence of noise is achieved by using sufficiently large statistical ensembles of realizations \( I_k(t) \) and smoothing the covariance estimates and the recovered Doppler-velocity profiles \( v_r(z) \). We do not consider the influence of the frequency chirp on the final results; that is, we assume that the chirp effect is negligible.

The results from the simulations show that algorithms (14a) and (14b) allow one to retrieve the profile of \( v_r(z) \) with a high spatial resolution but on the basis of a sufficiently large number of laser shots to reduce the error due to speckle noise. Thus the speckle noise limits the temporal resolution, i.e., the ability to observe rapid changes in atmospheric conditions. Because of the recurrence character of the algorithms, the noise-induced random error is accumulated and increases with the increase of \( z - z_o \). The step of the increase is equal to the laser pulse length and consequently is longer at longer sensing pulses. However, in this case the noise-error increment per step is higher. Therefore one should use pulses with optimum length such that the increase of the noise error with \( z - z_o \) is slow. Filtering the covariance estimate \( \hat{\text{Cov}}(t, \theta) \) or the retrieved profile \( v_r(z) \) of \( v_r(z) \), or both, is an effective way to suppress the speckle-noise influence and thus to reduce the number of laser shots required to achieve a prescribed accuracy. Such an approach leads to improvement in the temporal resolution of retrieving \( v_r(z) \) but is accompanied by some lowering of the spatial resolution. Hence the effective width of the filter window should have an optimum value, thus ensuring considerable noise suppression at an acceptable high spatial resolution.

In Figs. 6(a) and 6(b) we have represented the profiles of \( v_r(z) \) restored by use of relations (14a) and (14b), respectively. These profiles correspond to the model of \( v_r(z) \) described by Eq. (16). The laser wavelength and pulse length assumed are \( \lambda = 10.6 \, \mu m \) and \( l_p = 1200 \, \mu m \), respectively. The number of simulated laser shots (signal realizations) is \( N = 2000 \). A 8\Delta t_o-\( (8\Delta z_o) \)-wide-window moving average smoothing of \( \hat{\text{Cov}}(t, \theta) \) and \( v_r(z) \) is also performed. So the achievable resolution cell is \( R \approx 8\Delta t_o = (120 \, m) \). It is shown in Figs. 6(a) and 6(b) that the speckle-noise effect accumulates with the increase in distance \( z - z_o \). Also, it is noticeable that at longer distances \( (z - z_o > 3 \, \text{km} \) in the present case), algorithm (14a) is more affected by the speckle noise. At shorter distances from \( z_o \) (\( z - z_o \) several pulse lengths) both algorithms lead to acceptable accuracy of retrieving \( v_r(z) \) even at narrower filter window and considerably smaller number of realizations (e.g., \( N = 500 \)). The last property of the discussed algo-
The dotted curves in Figs. 6(a) and 6(b) represent the profile of $v(r)$ corresponding to the same model of $v(z)$ [Eq. (16)] but obtained by using the PP algorithm. The number of sampling points employed for averaging along the line of sight is $M = 20$. Thus the resolution cell achievable in this case is $R_{pp} = c(M \Delta t_o + \tau)/2 = c(20 \Delta t_o + \tau)/2; \Delta t_o = 0.1 \mu s, \tau = 4 \mu s$. To reduce the speckle-noise effect, the covariance estimate $\text{Cov}(t, \theta)$ is obtained by additional averaging over $N = 500$ realizations of $I(t)$. It can be seen that the resultant noise error is negligible, but there is a noticeable error due to the averaging along the line of sight and the finite pulse duration, i.e., as if the profile $v(r)$ is obtained by a $R_{pp}$-wide-window moving averaging of $v(z)$.

Profiles of $v(r)$ corresponding to the model of a wind vortex [Eq. (18)] and restored by the use of formulas (14a) and (14b) are shown in Figs. 7(a) and 7(b), respectively. They concern the case of rectangular pulse (with $\tau = 0.2 \mu s$), but the same are in practice the results for $\lambda = 10.6 \mu m$ and $\tau = 4 \mu s$ (see Ref. 25). In order to avoid excessive accumulation of the noise, the wind vortex area begins from the dead-zone end ($z = z_o$) and is covered by only several pulse lengths [$z - z_o - (2 - 3) \nu \tau/2$ when $\tau = 200 ns$ and $\lambda = 2 \mu m$, and $z - z_o \ll \nu \tau/2$ when $\tau = 4 \mu s$ and $\lambda = 10.6 \mu m$]. Thus $v(z)$ is accurately retrieved on the basis of $N = 1000$ realizations $I(t)$ after filtering $\text{Cov}(t, \theta)$ by a moving average with a $6 \Delta t_o$-wide window ($R = 9 m$) when Eq. (14a) is used and a $4 \Delta t_o$-wide window ($R = 6 m$) when Eq. (14b) is used. The same profile of $v(z)$ (dashed curve) and the profile obtained with the PP algorithm (dotted curve) are given for comparison.

![Fig. 6](image_url) Alternating radial wind velocity profiles $v_r(z)$ restored in the case of rectangular pulse (with $\lambda = 10.6 \mu m$ and $\tau = 4 \mu s$) by use of (a) algorithm (14a), (b) algorithm (14b), and (c) algorithm (14a) and the lidar dead-zone scanning technique. The sampling interval employed is $\Delta t_o(\Delta z_o) = 0.1 \mu s$ (15 m). The number of signal realizations employed is (a) and (b) $N = 2000$ and (c) $N = 500$. $\text{Cov}(t, \theta)$ and $v_r(z)$ are smoothed by a moving average with (a) and (b) $8 \Delta t_o$- $(8 \Delta z_o)$-wide window and (c) $4 \Delta t_o$- $(4 \Delta z_o)$-wide window. The original profile of $v(z)$ [described by Eq. (16)] is given for comparison by the dashed curve. The dotted curves in (a) and (b) represent the result obtained with the PP algorithm.
by using a PP estimator at signal realizations case of near-rectangular (rectangularlike) pulses. For this purpose we have considered the following rectangular approximation of real pulse shapes, it is of interest to prove whether the approach developed here for rectangular pulses will lead to an acceptable recovery of \( v(z) \) in the case of near-rectangular (rectangularlike) pulses.

\[
\hat{R}_{pp}(\Delta z, \Delta t) = \frac{1}{M} \sum_{i=1}^{M} \left( R_{pp} \left( \Delta z, \Delta t + \Delta t_i \right) - \hat{R}_{pp}(\Delta z) \right)^2
\]

For the case of uniform aerosol velocity and concentration within the statistical volume, as may be expected, at relatively small rise (decay) time \( \tau_r \), an accurate restoration of \( v(z) \) is achieved. The restoration error increases with the increase in \( \tau_r \) but retains an acceptable value even at pulse shapes that have near-triangular form. The profiles of \( v(z) \) obtained by use of the PP algorithm are in general more distorted [with respect to the true profile \( v(z) \)] than those obtained with the algorithms developed here. Only at relatively large values of \( \tau_r \), when the rectangularlike pulses become in fact triangularlike ones do the two types of distortions become comparable.

The simulations conducted show that the approach developed here of retrieving \( v(z) \) permits one to reach a high spatial resolution without specifically decreasing the laser radiation wavelength, the pulse duration, and the sampling interval. At the same time, appropriate smoothing and lidar dead-zone scanning techniques as well as powerful lasers with high pulse-repetition rates should be used to ensure acceptable temporal resolution.

6. CONCLUSION

The inverse mathematic techniques developed in this study can be used successfully for retrieving high-range-resolution Doppler-velocity profiles on the basis of data obtained by coherent heterodyne Doppler lidars with (relatively long) rectangular and rectangularlike sensing laser pulses. These techniques allow one to achieve resolution scales that are considerably smaller than the pulse length. The techniques generalize some known techniques, including the PP algorithm, of determining Doppler-velocity profiles that are strictly valid only in the case of uniform aerosol velocity and concentration within line-of-sight intervals of the order of the pulse length.

The speckle noise is the main disturbing factor that leads to the appearance of noise in the final results. The use of a large number of laser shots provided by powerful lasers with high pulse-repetition rates is a way to suppress the noise and obtain accurate results. To avoid a too-large statistical volume, and thus to accelerate the measuring procedures (to improve the temporal resolution), one can essentially reduce the speckle-noise effect by filtering the autocovariance estimates and the recovered noisy profiles as well as by using the lidar dead-zone scanning technique.

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REFERENCES