On the use of non-Fourier eigensignals for measuring short optical pulses by photomultiplier tubes

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Abstract. In this paper we show that the envelopes of short optical pulses detected by conversion of photomultiplier tubes—secondary electrons into single-frequency decayed oscillations may be presented by a set of orthogonal non-Fourier eigensignals. The enhanced temporal resolution estimated is of the order of (10–1) ps. A calculation of an optical receiver based on the commercial multianode microchannel plate photomultiplier—photomultiplier tubes is given. The method may be successfully applied in such areas as picosecond lidars, fibre optics, time-resolved spectroscopy etc.

1. Introduction

The measurement of the envelopes of short optical pulses is an essential problem for many recent applications in time-resolved spectroscopy, fibre optics, high resolution remote sensing etc. [1–3]. Different methods are known for optical shape detection—photomultiplier tubes in a single quantum regime of resolution 50–20 ps [4, 5], streak-cameras of resolution down to 0:6 ps [6, 7], optical Kerr-shutters of resolution 3–5 ps [6, 8], multiple-shot optical correlators with a resolution of the order of several femtoseconds [9]. One of the requirements for the advanced electro-optical systems operating with short pulses, is to combine the high temporal resolution with a high amplitude resolution, high sensitivity, short integration time, wide dynamic range and low cost.

Photomultiplier tubes (PMTs) offer a relatively better compromise of the above parameters. Some problems arise when it is necessary for the illuminating intensity to be attenuated to maintain single counts per measuring event and, thus, the signal energy is not fully used which causes the increase of the integration time. It is of interest to improve the temporal resolution of the PMTs limited by the time-jitter, time-broadening etc., which are determined mostly by the photocathode and the multiplying structures of the PMTs. Their quality is presently close to the physical limitations.

A new approach to improve the temporal resolution of a PMT was developed in [10–14] for the model of a photodetecting system, comprising a PMT, a set of resonators, recording equipment and retrieving algorithms. Optical receivers of this type possess many essential advantages combining both high temporal and amplitude resolution, linearity at arbitrary intensities without the non-effective overlapping regime, high sensitivity to the PMT, sampling by low-speed analogue-to-digital (A/D) converters etc. One of the conclusions in these works [13] is that the resolution may be increased at the same PMT-jitter and time-broadening without the necessity of improving the photocathode and multiplying structures. The main
goal of the present work is to show how to use the method [10–14] to realizesuch an
idea and to estimate the resolution which is to be expected in an experiment. For this
purpose an inverse problem for the photodetection in such a system is formulated,
based on the set of orthogonal, non-Fourier eigensignals of frequencies within the
PMT bandwidth. As a consequence, weak and short optical pulses could be
measured with a high amplitude and temporal resolution, avoiding the gating
technique in the case of optically isolated pulses. It is shown that a resolution of the
order of $10^{-1}$ ps may be achieved, when a commercial multianode microchannel
plate photomultiplier tube (MCP-PMT) is used.

2. Formulation of the problem

2.1. Linear set of equations in respect to the optical envelope

The model of the optical receiving system of interest was described and analysed in
[10–14]. It comprises a PMT, a set of resonators and a signal-recording and
processing system. The secondary electrons in the PMT, possibly of multianode
type, interact with the resonators, exciting single-frequency decayed oscillations of
frequencies $\omega_q$, $q=1 \ldots Q$ and relaxation time $\tau_r$. The output signal is a result of
interference of oscillations, excited by all the electrons during the observation (gating)
interval $t_g$ of repetition period $T_r < \tau_r$ within the integration time. The input
optical flux (profile) may be described with a resolution $\Delta \vartheta$ by the number of photons
$N_i = N(\vartheta_i) + n_0$ arriving with some internal delay $\vartheta_i = l \Delta \vartheta$, $l=1 \ldots L$ within the
interval $t_g$, where $n_0$ is the mean number of photons per resolution interval $\Delta \vartheta = t_g / L$
(averaged over the observing interval); $N(\vartheta_i)$ is the centred, time-resolved profile,
describing the fine temporal structure of the optical envelope. The quantum
efficiency $\eta$ can be introduced by substitution $N_i \rightarrow \eta N_i$. The amplified and detected
output signals are further sampled and used in inverse algorithms to retrieve the
optical profile $N_i$.

Following [14], the mean signals $S(\omega_q, t)$ on the resonator outputs can be
expressed (after averaging in a single pulse using the model of noises in the PMT) by
temporally decayed oscillations of frequencies $\omega_q$, $q=1 \ldots Q (L = Q$ for amplitude
detection and $L = 2Q$ for quadrature detection of $S(\omega_q, t)$ [14]:

$$S(\omega_q, t) = C_q A(\omega_q) \exp \left\{ - (\beta - j \omega_q) t \right\},$$

(1)

where $C_q$ is the calibration constant, $\beta = 1 / \tau_r$. The statistical properties of $S(\omega_q, t)$ are
investigated theoretically and experimentally in details in [10, 14]. The amplitudes
$A(\omega_q)$ in (1) are given by

$$A(\omega_q) = H(\omega_q) \left[ \sum_{l=1}^{L} N(\vartheta_l) \alpha_{lq} + n_0 V(\omega_q) \right],$$

(2)

where $H(\omega)$ is the normalized transfer function of the system PMT—resonator [14],
$C_q = 1$ may be accepted, $\alpha_{lq}$ are the matrix coefficients, depending on the frequency $\omega_q$
and the position $\vartheta_l$ of the resolution interval within the gating interval $t_g$

$$\{ \alpha_{lq} \} = \{ \exp (j \omega_q \vartheta_l) \}.$$

(3)

$V(\omega_q) = \Sigma_{l=1}^{L} \alpha_{lq}$. The amplitudes $A(\omega_q)$ in (2) contain (by the matrix $\alpha_{lq}$) information
about the envelope samples $N(\vartheta_l)$ and $n_0$ within the observing interval. The
covariance function $R_q(\tau)$ of $S(\omega_q, t)$ is now given by

$$R_q(\tau) \propto H^2(\omega_q) \frac{\exp \left\{ - 2 \beta \tau \right\}}{2 \beta} \cos (\omega_q \tau).$$

(4)
Figure 1. Frequency spectra $S_q(\omega)$ of the orthogonal signals $S(\omega_q, t)$ of spectral width $\delta \omega_q$ within the bandwidth $\Delta \omega_r$ of the system PMT-resonators. $H(\omega)$ is the transfer function, $\Omega_q$ is the frequency difference between the adjacent resonators.

As seen from equations (2) and (4), the transfer function $H(\omega)$ imposes limitations on the resonator frequencies $\{\omega_q\}$ to be chosen only within the bandwidth $\Delta \omega_r$ of the system transfer function $H(\omega)$ [14].

Now, the frequency spectra $S_q(\omega)$ of the signals $S(\omega_q, t)$ may be determined using the covariance functions $R_q(t)$. The spectra $S_q(\omega)$ (figure 1) are of narrow-bandwidth type with mean frequencies $\omega_q$ and bandwidths

$$\delta \omega_q = 2/\tau_{\omega} < \Delta \omega_r,$$

(5)
depending on the relaxation time $\tau_{\omega}$. If the frequency differences between the adjacent resonators $\Omega_q = \omega_{q+1} - \omega_q$ is chosen according to the condition $\Omega_q > \delta \omega_q$, $q = 1 \ldots Q$, the spectra $S_q(\omega)$ and $S_{q+1}(\omega)$ of the signals do not overlap. Therefore, the orthogonality of $S_q(\omega)$ and $S_q(\omega)$ in the frequency range $0-\Delta \omega_r$ can be expressed by the integrals

$$\frac{2}{[R_q(0)R_q(0)]^{1/2}} \int_0^{\infty} S_q(\omega) S_{q'}(\omega) d\omega = \begin{cases} 1, & q = q', \\ 0, & q \neq q', \end{cases}$$

(6)

where the upper limit $\infty$ may be replaced by $\Delta \omega_r$. Under these conditions the signals $S(\omega_q, t)$, $q = 1 \ldots Q$ form an orthogonal basis of eigenfunctions of relatively long duration $\tau_{\omega} > t_{\omega}$.

By sampling the amplitudes of detected signals $S(\omega_q, t)$, a basis of independent amplitudes $A(\omega_q)$, $q = 1 \ldots Q$ can be measured. Expressions (2) are reduced now to a set of linear equations with respect to the centred time-resolved optical envelope $N(\theta_i)$,

$$B(\omega_q) = \sum_{l=1}^{L} a_{\omega_q} N(\theta_l),$$

(7)

where $B(\omega_q) = A(\omega_q) - n_0 V(\omega_q)$ are the centred amplitudes.
If \( A(\omega_q), q = 1 \ldots Q \) are measured, \( B(\omega_q) \) may be calculated (the mean flux \( n_0 \) is determined independently—see Section 3.2. and [14]). Solving the system (7) with respect to the time-resolved optical envelope we obtain

\[
N(\theta_t) = \sum_{q=1}^{Q} \left( \frac{D_{q\ell}}{D} \right) B(\omega_q),
\]

where \( D \) is the determinant of the matrix \( \{a_{i\ell}\} \) and \( D_{q\ell} \) are the consequent co-factors [15].

2.2. Preliminary discussion

At \( D \neq 0 \) there is a single solution of the system (7), written in the general form (8), where the centred time-resolved envelope \( N(\theta_t) \) is presented by the centred amplitudes \( B(\omega_q) \) of the orthogonal set of eigensignals \( S(\omega_q, t) \) on the resonator outputs.

It is of interest how to provide a resolution better than those in the standard regimes of a PMT based on the estimates of the limiting resolution of the method [12], when the frequencies \( \{\omega_q\} < 2\pi/t_s \) and, therefore, the eigenfunctions \( S(\omega_q, t) \) are of non-Fourier type. For the case of Fourier resonator frequencies, the resolution is of the order of the resolution in the analog regime of a PMT [14].

It is essential to analyse the opportunity and conditions, when the optical envelope within observing intervals of order of \( 10 - 1 \) ps may be presented by a set of \( Q > 1 \) orthogonal signals of frequencies not higher than the PMT bandwidth. The influence of noises is substantial for the choice of \( \{\omega_q\} \). Because of the technical limitations to strobe by intervals shorter than \( t_{s\min} \approx 1 \) ns, the determinant \( D \) must be analysed at \( Q \gg 1 \), if higher resolutions are required. At gating intervals close to \( t_{s\min} \) the effect of the gating function on the pulse shape has to be taken into account at the final retrieving of \( N_t \). As shown below the tolerable number of \( Q \) does not exceed ten, if the basis is of non-Fourier type or it is of the order of the number of anodes in the commercial multianode PMTs. Because of that, we will pay special attention to the measurement of short, temporally isolated optical pulses of duration \( \ll t_{s\min} \) when the gating may be avoided.

3. Temporal resolution for the case of isolated short pulses

3.1. Effects of noises on the choice of resonator frequencies

If the locality of the time-resolved optical profile \( N(\theta_t) \) is noise-disturbed [14], the matrix coefficients will not be well distinguished and the solution (8) will be incorrect. The locality is provided, if the resolution \( \Delta \theta \) exceeds the limiting resolution, determined in [12]

\[
\Delta \theta > \Delta \theta_{\text{min}} \approx \delta T_J / \sqrt{N_t} \approx \delta T_J / \sigma_n,
\]

where \( \bar{N}_t \approx n_0 = \sigma_n^2, \sigma_n^2 \) is the Poisson flux variance within the resolution interval, \( \delta T_J \) is the PMT time-jitter. The limiting resolution might be achieved, if \( t_s / \Delta \theta_{\text{min}} \) independent signals \( S(\omega_q, t) \) could be created at \( D \neq 0 \).

An additional condition for the frequency differences \( \Omega_q \) is the requirement for the statistical distinction of the amplitudes \( A(\omega_q) \), which is defined by both the variance \( \sigma_{A,q}^2 \) of \( A(\omega_q) \) [12, 14] and the sampling noise variance \( \delta A^2 \) of the \( A/D \) converters or

\[
\langle [A(\omega_{q+1}) - A(\omega_q)]^2 \rangle \approx K_0^2 (\sigma_{A,q+1}^2 + \sigma_{A,q}^2 + 2 \delta A^2),
\]
Here we will assume a statistical independence of $N(\theta_i)$ for different $l$ or $\langle N(\theta_i)N(\theta_l) \rangle = \delta N^2 \delta_{ll'}$, where $\delta N^2$ is the variance of $N(\theta_i)$. Averaging (10) over a set of profiles $N(\theta_i)$, the following estimate for $\Omega_q$ is obtained:

$$\Omega_q = \Omega_0 \simeq \frac{K_0}{t_s \sqrt{N_t}} \left( 1 + \frac{\delta A^2}{\sigma_{A,q}^2} \right)^{1/2},$$

where $N_t$ is the total energy. Because of the narrow bandwidth $\delta \omega_q$ of $S(\omega_q, l)$ and the use of low speed, high resolution $A/D$ converters (typically 12–16 bits, 1 kHz) the sampling noises may be neglected ($\delta A^2 \ll \sigma_{A,q}^2$) [13]. Now at $\Omega_0 \gg \delta \omega_q$ (see Section 3.3.) and $N_t \approx n_0 L$ the frequencies $\{\omega_q\}$ are given by

$$\omega_q \approx \omega_1 + (q-1)\Omega_0; \quad \Omega_0 \approx 10/(t_s \sqrt{N_t}),$$

if $K_0 \approx 3$. The lowest frequency $\omega_1$ may be equal to $\Omega_0$ and, thus, $\omega_q = q\Omega_0$. Now, if $\{\omega_q\}$ are chosen according to equation (12), the effect of noises may be neglected in the first-order approximation.

3.2. Equivalent gating

In some cases when short pulses are measured the gating may be effectively avoided, applying the method, developed below. Let us assume (figure 2) that an optical pulse of mean width $\bar{\tau}_p$ is observed after some delay $t_d$ with respect to the triggering pulse, the pulse energy is localized within a limited temporal interval $\delta \tau$, of order of several $\bar{\tau}_p \ll t_{\text{min}}$ and the optical energy outside of the interval $\delta \tau$ may be neglected. Pulses of this type can be accepted as temporally isolated.

Let us choose two additional resonators of frequencies $\omega_0$ and $\omega_0'$ satisfying the conditions: $\omega_0 \ll \omega_1$, $\omega_0' \approx \omega_0 + \Omega_0 \ll \Delta \omega_p$. Assuming the model of a rectangular pulse shape of width $\bar{\tau}_p$ [11], equations (7) for the frequencies $\omega_0$ and $\omega_0'$ are reduced to

$$A(\omega_0) = N_t, \quad A(\omega_0') = N_t \sin \left( \frac{\omega_0 \bar{\tau}_p}{2} \right) / \left( \frac{\omega_0 \bar{\tau}_p}{2} \right).$$

Now, the estimate of $\bar{\tau}_p$ (when $\bar{\tau}_p \ll 2\pi/\omega_0$) is given by

$$\bar{\tau}_p \approx 5 \frac{1 - A(\omega_0')}{A(\omega_0)}. \quad (14)$$

The pulse delay $t_d$ may be easily measured at some third frequency $\omega_0'' \ll 2\pi/t_d$ by a phase method [16]. At known estimates of $\bar{\tau}_p$ and $t_d$ we could further define (figure 2)

![Figure 2](image.png)

**Figure 2.** To the definition of the equivalent gating interval $\delta \tau > \bar{\tau}_p$ at a delay $t_d$, where $\bar{\tau}_p$ is the mean pulse width and $t_d > t_d$ is the pulse delay.
two new estimates of $\vartheta_s$ and $t_p$ by the inequalities $\vartheta_s > \bar{\vartheta}_p$ and $t_P < t_s$ and so to state that the measured pulse is localized within the equivalent gating interval $\vartheta_s$ with a nearly 100% probability. The estimate $N_i$ of the pulse energy is not affected by such a procedure. Further, we can use the estimated value of $\vartheta_s$ in the above expressions instead of $t_s$. The frequency range of the resonators is defined by $\omega_q \leq \Delta \omega_r \leq 2\pi/\vartheta_s$. At $\Delta \omega_r < 2\pi/\vartheta_s$, the entire bandwidth of the PMT may be used. The above approach is equivalent to the gating of optical signals without the use of strobing technique. It results in decreasing the required number of resonators, because the short pulses of duration $\ll t_{\text{min}}$ may be well localized, avoiding the use of samples of zero magnitudes. Equations (7) may now be rewritten for the optical profile $N(\vartheta_s)$ within the equivalent gating interval $\vartheta_s$, and so the resolution will be equal to $\Delta \vartheta = \vartheta_s/\Omega$. As seen from equation (13), the mean width $\bar{\vartheta}_p$ and, therefore, $\vartheta_s$ may be determined with a high accuracy, when $\bar{\vartheta}_p \geq 1 \text{ ps}$ and $\Omega_0 \sim 1 \text{ GHz}$. Thus, the number of resonators required may be decreased to a reasonable value.

3.3. Solutions for different number of resonators

The determinant $D$ (see equation (8)) is of Vandermonde type [15]. It can be shown that $D \neq 0$ and tends to zero at $Q \to 1$. The fast decrease of $D$ imposes essential restrictions on the number of resonators at a given observing interval $\vartheta_s$. In such cases $D \neq 0$ is not the best criteria for the solution of equation (7). Better estimates may be obtained by an analysis of the solutions at different $Q$ and by a proper choice of the frequencies $\{\omega_q\}$ [12] for a given $\Delta \omega_r$, varying the equivalent interval $\vartheta_s$.

In the following calculations some commercial multianode MCP–PMTs with $\Delta \omega_r \geq 1 \text{ GHz}$ are assumed [17]. It is of interest to evaluate the maximum number $Q_m$ of resonators at which the system (7) may be correctly solved as a function of $\vartheta_s$. The theoretical dependence of the resolution $\Delta \vartheta = \vartheta_s/Q_m$ as a function of $\vartheta_s$ is given in figure 3(a) for quadrature and in figure 3(b) for amplitude detection of output signals. The consequent number of $Q_m$ is given too. As seen, $Q_m \leq 7$ for a quadrature detection and does not exceed 10, when an amplitude detection is used. The resolution $\Delta \vartheta = \vartheta_s/Q_m$ is better at shorter pulses, when $Q_m$ is lower. As an example, if $\vartheta_s = 1 \text{ ns}$, we obtain $Q_m = 10$. If $\Omega_0 = 100 \text{ MHz}$ the frequencies in equation (12) will be $\omega_q = q 100 \text{ MHz}$. For pulse durations of the order of 1 ps the maximum number of resonators is $Q_m = 3$ (amplitude detection) and the expected resolution is 0.3 ps, i.e. it is approximately the same as in the best streak-camera-tubes. It should be noted, that the pulse shapes of duration $\vartheta_s$ will be determined by the same number of samples $N(\vartheta_s)$ as in the streak-cameras.

Therefore, the final resolution is provided by two steps. At the first one using the three frequencies $\omega_0$, $\omega_1$ and $\omega_2$ the position of the measured pulse is localized, determining the equivalent gating interval $\vartheta_s$. At the second step the main set of frequencies $\{\omega_q\}$ is used. As an example, for 10 ps resolution (see figure 3(b)) a set of $Q_m = 5$ frequencies may be used (totally eight frequencies $< 1 \text{ GHz}$). The resolution by an amplitude detection is practically the same as by the quadrature detection and may be recommended to simplify the entire system.

As seen from equation (12) the frequency difference $\Omega_0$ depends on the signal level. As a result, the pulse energy $N_i$ should exceed some minimum value $N_{\text{min}}$ in order to hold the frequency differences $\Omega_0$ less than $\Delta \omega_r/Q_m$ at a given $\vartheta_s$, where

$$
N_{\text{min}} \simeq (10^2 Q_m^2)/(\vartheta_s^2 \Delta \omega_r^2).
$$

(15)
Figure 3. Dependence of the resolution $\Delta \theta = \theta_s / Q_m$ of the retrieved profile $N$ on the equivalent gating interval $\theta_s$ for quadrature (a) and amplitude (b) detection of the output signals $S(\omega, t)$. The consequent maximum tolerable number of resonators $Q_m$ is denoted.

4. Discussion: Comparison with photomultiplier tube

In terms of eigenfunctions one can state that, if the resolution of a standard PMT is equal to 50 ps, the tolerable number of eigenfunctions describing the optical envelope on PMT output within the interval of 50 ps is equal to one and it is zero at higher resolutions. As seen, when the new eigenfunctions are used, the temporal resolution estimates of the method (10 - 1 ps) are better than the resolution of a PMT in the single quantum mode (50 - 20 ps). By the new approach presented, the tolerable number of eigenfunctions $S(\omega, t)$ within the same bandwidth may be greater than one, resulting in a better resolution at the same fluctuating parameters of
the PMT—time-broadening, time-jitter etc. Therefore, the approach developed here is one of the ways [13] to improve the resolution of recent PMT without the need to create new types of photocathodes and multiplying structures.

It is essential to note that the resolution depends on the optical intensity [13]. The dependence of \( N_{\text{min}} \) as a function of \( \Delta \theta \) at a PMT bandwidth of 1 GHz and using the data for the tolerable number of eigenfunctions is plotted in figure 4. The minimum number of photoelectrons needed varies within the range of \( 10^4 \)–\( 10^9 \), consequently giving low signal energies. The high sensitivity of the PMT is not disturbed when the new method is used. As a rule, the regimes of the PMTs at these energies and the resolutions are of analog or overlapping types. For comparison, the resolution \( \Delta \theta_{AD} \approx 1/\Delta \omega_c \) of PMT in analog regime [18, 13] is much lower than by the method developed (\( \Delta \theta \ll \Delta \theta_{AD} \)). It should be noted that the linear performance in all regimes was experimentally demonstrated in [10]. A shorter integration time may be provided, because it is not necessary to attenuate the input flux as in the single regime of the PMTs. As seen, the high temporal resolution (10–1 ps) of the method is well combined with the highest sensitivity of the PMT and, further, because of the use of low speed, high capacity A/D converters—with a high amplitude resolution and dynamic range.

The effect of the measuring system was partially taken into account in equation (11) by the sampling variance \( \delta A \). The actual resolution is restricted by the measuring jitter \( \delta T_m \) of the entire system. If \( \Delta \theta \ll \delta T_m \) [14], the final resolution will be determined by \( \delta T_m \). The effect of \( \delta T_m \) may be decreased integrating over a number of repetition periods.

5. Calculation procedure of the optical receiver

In this Section we will demonstrate the determination of the main parameters of an optical receiver, based on the method proposed. As an example, a commercial ‘Hamamatsu’ 16-anode MCP-PMT of type R 1712 [17] is taken with a rise time 0.27 ns and a transit time 0.58 ns. The time-jitter is defined by the transit time-spread of the PMT. It does not exceed 100 ps for the tubes of similar series [17]. As shown in
[14], the bandwidth $\Delta \omega$, of the system PMT resonator is of the same order as the PMT bandwidth ($\Delta \omega \approx 1.23$ GHz [17]). Thus, all the resonator frequencies $\omega_0$, $\omega'_0$, $\omega''_0$ and $\{\omega_q\}$ should be localized within the range of $0 \leq 1.23$ GHz.

A scheme of the optical receiver is shown in figure 5. Each resonator is fed to one of the anodes for a better isolation from the other channels. The illumination is assumed uniformly over the entire photocathode. The resonator outputs are fed through amplifiers and amplitude detectors (AD) to the multiplexer (MP) and then to the A/D converter and signal processor for retrieving the pulse shape. The frequencies, calculated by the results in figure 3(b) are as follows: $\omega_0 = 1$ MHz; $\omega'_0 = 1.2$ GHz; $\omega''_0 = 100$ MHz; $\Omega_0 = 100$ MHz; $\omega_1 = 200$ MHz; $\omega_q = [200 + (q - 1) 100]$ MHz.

The lowest frequency resonator ($\omega_0 = 1$ MHz) can be of quartz type as that used in the experiments, presented in [10]. All the other resonators may be produced by an integrated surface-acoustic wave (SAW) technology [19]. An essential simplification here is the non-critical requirement to the values of the relaxation times. As shown in [14] a quality factor of the resonators of the order of $10^3$–$10^4$ is tolerable and, therefore, $\tau \geq 10^{-4} - 10^{-5}$ s. The measurement and calculation procedures are summarized below:

1. Calibration of the receiver as shown in [14] using pulses of well-known characteristics.
2. Determination of the pulse energy $N_t$, equivalent gating cell $g_e$ and the time delay $t_p$ (see equations (13) and (14)). It may be noted, that the time delay $t_p$ does not enter equations (7, 8) and may not be measured, in principle.
3. At known values of $g_e$, using the selection chart (figure 3), one can determine the maximum tolerable number of resonators $Q_m$ and then to calculate the minimum signal energy $N_{min}$ at a given bandwidth $\Delta \omega$. If $N_t > N_{min}$, the temporal resolution after the solution of the linear system (equation (7)) will be $\Delta \delta = g_e/ Q_m$. Further, the estimates of $n_0 = N_t/ Q_m$ and the variance $\sigma_n$ may be determined as well as the frequencies $\{\omega_q\}$.
4. According to equation (8), the linear system may be solved, using the measured amplitudes $A(\omega_q)$. As a result, the time-dependent profile $N(\delta)$ may be calculated and then the full pulse shape can be retrieved using the values of $n_0$ and $N(\delta)$. The accuracy of the estimated envelope may be easily calculated, assuming a Poisson distribution of the number of photoelectrons and using the known estimates of $N(\delta)$, $n_0$ and $\sigma_n$, discussed above.
If $\theta_s$ is chosen too large, the above procedure may be repeated, using a shorter value of $\Omega_0$ corrected by the retrieved on the first step profile $N(\theta)$.  
(5) If $N_r < N_{min}$ the resolution will be lower. In this case one has to calculate the tolerable value of $\Omega_0$ using equation (11) and the number of resonators $Q \geq \Delta \Omega / \Omega_0 < Q_m$. Further, a new set of frequencies (within the set previously chosen) must be defined satisfying the condition $N_r > N_{min}$. It is evident, that if $N_r < N_{min}$ and $Q = 1$, it is not possible for the pulse shape to be resolved within the interval $\theta_s$.

6. Conclusions

A set of orthogonal non-Fourier eigensignals of frequencies not higher than the PMT bandwidth is introduced and analysed to describe the envelopes of short optical pulses on the receiver output, based on the photodetection method by conversion the PMT-electron trains into single frequency decayed oscillations. It is theoretically shown that the temporal resolution is of the order of $(10-1) \text{ps}$ or the resolution better than the corresponding of a PMT in the standard regimes at the same photocathode and multiplying structures may be provided. The observing interval for the case of temporally isolated pulses may be determined without gating by the use of the same technique. A calculation of the main parameters using commercial multianode MCP-PMTs is given. The optical receivers of this type offer one of the best compromises between high temporal and amplitude resolution, low cost, short integration time, sensitivity, dynamic range and may be successfully applied in such areas as picosecond lidars, fibre optics and time-resolved spectroscopy.

References