A dual interpretation of experimental data concerning the propagation of laser light through tissue-like turbid media

I. Bliznakova*, L. Gurdev, T. Dreischuh, O. Vankov, D. Stoyanov, L. Avramov
Institute of Electronics, Bulgarian Academy of Sciences
72 Tzarigradsko chaussee Blvd., 1784 Sofia, Bulgaria

ABSTRACT

The propagation is investigated of a continuous laser beam through homogeneous tissue-like turbid media such as diluted emulsions of Intralipid or milk having presumably sharply forward directed Henyey-Greenstein or Gaussian indicatrices. The cross-sectional radial distributions of the detected forward-propagating light power at different depths along the beam axis in each medium of interest are experimentally determined. The detected-power spatial distribution, for both the types of indicatrices, is also described analytically by a solution of the radiative transfer equation in the so-called small-angle approach. The experimental results are consistent with the analytical expressions obtained that are shown to allow one to estimate the extinction ($\alpha_s$), reduced-scattering ($\alpha_{rs}$) and absorption ($\alpha_a$) coefficients and the $g$-factor of the investigated media. The values obtained of $\alpha_s$, $\alpha_{rs}$ and $g$ of the dilutions of concern are quite reasonable and behave, depending on the dilution turbidity, in a way observed formerly in other similar experiments. The comparative analysis of the estimated characteristics of the dilutions shows that in the case of Henyey-Greenstein indicatrix we have a smaller value of the $g$-factor and larger value of $\alpha_{rs}$ with respect to the case of Gaussian indicatrix. At equal $g$-factors, in the former case we shall have a narrower forward-propagating scattered-light beam with higher on-axis intensity as compared with the latter case.

Keywords: Propagation of laser radiation, biological tissues, turbid media, laser beam propagation

1. INTRODUCTION

Optical tomography is an important hopeful investigation area that is expected to provide effective non-invasive and ionizing-radiation free methods and instruments for early diagnosis of serious human tissue diseases. The numerous variants of optical tomography being developed at present (e.g., 1-5) require the knowledge of the optical properties of tissues and the laws governing the radiative transfer within the investigated biological objects. The main optical characteristics of importance specifying the normal and abnormal tissues and conditioning the radiative transfer are the scattering $\alpha_s$ and reduced scattering $\alpha_{rs}=\alpha_s(1-g)$ coefficients, the absorption coefficient $\alpha_a$, the scattering indicatrix, and the anisotropy factor $g$. For optical radiation with wavelength $\lambda$ from 400 nm to 900 nm the biological tissues are turbid media characterized by strong scattering and considerably weaker absorption. Thus, the mean free path (MFP) of a photon in tissue will be determined in practice by $\alpha_s$, i.e., $\text{MFP} \sim \alpha_s^{-1}$. The value of the MFP in this case is usually less than 1 mm, which means that the multiple scattering effects will be essential and the radiative transfer equation should be used for quantitative analysis.

A purpose of the present study is to investigate experimentally the propagation of an optical (laser) beam through a homogeneous tissue-like medium and to determine the cross-sectional radial distribution of the detected forward-propagating light intensity at different depths along the beam axis. Similar in a sense thorough and precise experiments have been conducted in works 6-7, where the experimental results are compared with results obtained by Monte-Carlo simulations and some theoretical approaches. The agreement obtained is fairly good, but the problem remains unsolved about finding straightforward relations between the experimental-data values and behavior and the optical characteristics of the medium under investigation. Such relations would allow one to unambiguously and clearly determine the latter on the basis of the former ones. Therefore, another aim of the present study is to attempt achieving a comparatively simple analytical description, for relatively large or small “optical depths”, of the radial intensity distribution and the in-depth profile of the axial intensity of the beam. We suppose that such an aim is attainable by using the potential of the so-called small-angle approach to solving the radiative-transfer equation 8-10. This approach should be applicable to the media

*irbliznakova@abv.bg; phone +359 2 9795867; fax +359 2 9753201
under consideration in the work because they have sharp forward-directed indicatrices and not so high optical density. The final intention is to compare the experimental and analytical results and to outline ways for straightforward analytical estimation of the linear attenuation, reduced-scattering and absorption coefficients and the g-factor of the medium by using appropriate analytical expressions obtained for this purpose.

A central moment in the work will be as well the investigation of the influence of the kind of indicatrix on the spatial distribution of the detected light power and the estimated values of the optical characteristics of the medium. Two types of indicatrix, Henyey-Greenstein and Gaussian, will be considered, as more relevant to tissues and tissue-like turbid media.

2. SMALL-ANGLE APPROXIMATION OF THE DETECTION OF THE FORWARD PROPAGATING LIGHT POWER FOR GAUSSIAN AND HENYEY-GREENSTEIN SCATTERING INDICATRICES

For a stationary monochromatic radiation field inside a homogeneous and isotropic turbid medium without internal light sources, the radiative-transfer equation has the form

\[
\vec{s}.\text{grad}I(\vec{r};\vec{s}) = -\alpha_s I(\vec{r};\vec{s}) + \alpha_a \int i(\vec{s},\vec{s}')I(\vec{r};\vec{s}')d\omega',
\]

(1)

where \(I(\vec{r};\vec{s}) \text{ [Wm}^{-2}\text{sr}^{-1}]\) is the radiant intensity depending in general on the vector-coordinate \(\vec{r} = \{x, y, z\}\) and the directional unit vector \(\vec{s} = \{s_x, s_y, s_z\}\) \(s_x^2 + s_y^2 + s_z^2 = 1\); \(i(\vec{s},\vec{s}')\) is the scattering indicatrix from a direction \(\vec{s}'\) to the direction \(\vec{s}\); \(\int i(\vec{s},\vec{s}')d\omega' = 1\), and \(d\omega'\) is a differential element of solid angle; and \(\alpha_s = \alpha_s + \alpha_a\) [m\(^{-1}\)] is the total extinction (linear attenuation) coefficient in the medium, consisting of two components, the integral scattering coefficient \(\alpha_s\) and the absorption coefficient \(\alpha_a\).

The turbid medium of interest is assumed to occupy the semi-infinite space \(z > 0\) (Fig.1a); the axis \(0z\) of the coordinate frame is chosen to coincide with that of the incident laser beam. The task to be solved here based on Eq. (1) is first to determine analytically the radiant intensity \(I(\vec{r};\vec{s})\) of the forward propagating laser light (for \(s_z > 0\)) inside the medium. Then, an analytical expression should be obtained for the light power

\[
J(\vec{p}, z) = \int_A \int_{2\pi} R(\vec{p} - \vec{p}', z; \vec{s})d\omega'd\vec{p}'(2)
\]
detected by a circular optical receiver oriented antiparallelly to the beam axis; \(\vec{r} = \{\vec{p}, z\}\), \(\vec{p} = \{x, y\}\), \(\vec{p}' = \{x', y'\}\), \(R(\vec{p};\vec{s})\) is the receiver directional and aperture-transmittance diagram, \(A\) is the receiver aperture area, and \(2\pi + \) denotes integration over the positive unit hemisphere.

A way of analyzing the radiation transport equation (1), concerning the propagation of optical beams in turbid tissue-like media with sharply forward-directed indicatrices\(^{11-13}\), is the so-called small-angle approximation or approximation of large particles\(^{8-10}\). It allows one, under some reasonable assumptions, to obtain analytical results for \(I(\vec{r};\vec{s})\), and finally \(J(\vec{p}, z)\), ensuring relatively simple determination of the optical properties of a medium of interest.

We shall suppose here that the indicatrices of the investigated turbid media have respectively one of the following Gaussian or Heney-Greenstein forms:

\[
i_G(\vec{s}, \vec{s}') \equiv i_G(\{s_\perp, s'_\perp\}) = [2\pi(1 - g)]^{-1} \exp\left[-(s_\perp - s'_\perp)^2 / 2(1 - g)\right]
\]

(3a)

or

\[
i_{HG}(\vec{s}, \vec{s}') = i_{HG}(\mu) = [(1 - g^2)^2 / 4\pi](1 + g^2 - 2g\mu)^{-3/2},
\]

(3b)
where \( \bar{s}_\perp = \{ s_x, s_y \} \), \( \bar{s}'_\perp = \{ s'_x, s'_y \} \), \(||\) denotes module, \( \mu = \bar{s} \cdot \bar{s}' = \cos \theta \), \( \theta \equiv |\bar{s} - \bar{s}'| \equiv |\bar{s}_\perp - \bar{s}'_\perp| \) \(< 1 \) is scattering angle, and the anisotropy factor \( g_{HG} = \frac{\mu \mu_{HG}(\mu) d\omega'}{4\pi} \) in the latter case, and \( g_G \equiv \int_{0}^{\infty} \mu \mu G(|\bar{s}_\perp - \bar{s}'_\perp|) d\bar{s}'_\perp \) in the former case; \( d\bar{s}'_\perp = ds'_x ds'_y \).

The incident laser beam is considered as a collimated Gaussian one with field amplitude distribution

\[
u(\hat{r}, z = 0) = \nu_0 \exp(-\rho^2 / 2w^2),
\]

where \( w \) is the initial beam radius, and \( \nu_0 = \nu(\hat{r} = 0) \) is the initial “central” field amplitude. The corresponding radiance distribution \( I_0(\hat{r}; \bar{s}_\perp) = I(\hat{r}, z = 0; \bar{s}_\perp) \) is then given by \(^8\)

\[
I_0(\hat{r}; \bar{s}_\perp) = (k^2 w^2 I_0 / \pi) \exp(-\rho^2 / w^2 - k^2 w^2 s_\perp^2),
\]

where \( k = 2\pi / \lambda \), \( I_0 = |\nu_0|^2 \), \( \rho = |\hat{r}| \), \( s_\perp = |\bar{s}_\perp| \), and \( P_t = mw^2 I_0 \) is the total beam power. A radial intensity distribution in a widened cross section of the laser beam used in the experiments is given in Fig. 2b. It is seen that it has a Gaussian shape on the average.

It is expedient to assume that the receiving optical system has aperture radius \( E \) and Gaussian directional diagram with angle of view \( \gamma \), such that

\[
R(\hat{r}; \bar{s}_\perp) = \pi E^2 T \delta(\hat{r}) \exp(-s_\perp^2 / \gamma^2), \ T < 1,
\]

where \( \delta \) denotes delta function. As it is seen in Fig. 2c, the experimentally determined receiving directional diagram of the optical fiber employed in the measurements is well approximated by a Gaussian curve.

The large particle approach to solving Eq. (1) leads to a general result for \( I(\vec{r}; \bar{s}) \)^8. By using this result and Eq. (2), for the concrete models of \( iv(s_\perp) \), \( I_0(\hat{r}; \bar{s}_\perp) \), and \( R(\hat{r}; \bar{s}_\perp) \) given respectively by Eqs. (3a) and (3b), (5), and (6), we obtain the following asymptotic estimates for \( J(\hat{r}, z) \):

\[
J(\hat{r}, z) = P_t T E^2 \gamma^2 Q^{-1}(\chi, z, \gamma) \exp(-\rho^2 / w^2(z) - \alpha_{rs} z), \ \text{when} \ \alpha_{rs} z >> 1,
\]

and

\[
J(\bar{0}, z) = P_t T(E^2 / w^2) \exp(-\alpha_{rs} z)
\]

under the conditions \( \alpha_{rs} \gamma = \text{arctg}(z \gamma / w) << 2(1 - g) \) or \( \alpha_{rs} z << 1 \) corresponding to the cases of Gaussian or Henyey-Greenstein indicatrices; the corresponding values of \( \chi \) are \( \chi = \chi_G = \alpha_{rs} \) and \( \chi = \chi_{HG} = \alpha_{rs} (1 + g) / 3 \), and the quantity

\[
w(z) = [Q(\alpha_{rs} z, z, \gamma) / P(\alpha_{rs} z, \gamma)]^{1/2}
\]

is the \( e^{-1} \) half-width of the beam at a distance \( z \), where

\[
Q(\chi, z, \gamma) \equiv (1 / 3) \chi^3 (\chi z + 2 \gamma^2), \ \text{and} \ P(\chi, \gamma) \equiv 2 \chi^3 + \gamma^2.
\]

On the basis of relations (7), (8), and (9a-b) we obtain that

\[
\chi \equiv 6w^2(z) / z^3 \ \text{and} \ \alpha_{rs} = -(z_2 - z_1) \ln\left[\frac{J(\bar{0}, z_2) Q(\chi_{2, z_2, \gamma})}{J(\bar{0}, z_1) Q(\chi_{1, z_1, \gamma})}\right], \ \text{when} \ \alpha_{rs} z >> 1,
\]

and

\[
\alpha_{rs} = -(z_2 - z_1)^{-1} \ln\left[J(\bar{0}, z_2) / J(\bar{0}, z_1)\right]
\]

under the above-described conditions of validity of Eq. (8); \( J(\bar{0}, z) \) is the detected light power distribution along the beam axis, and \( z_1 \) and \( z_2 \) are two on-axis positions of the optical receiver. Thus, one could in principle estimate \( \alpha_{rs} \),
\( \chi \), and \( \alpha \) as well as \( \alpha_{rs} \) and \( g \) by measuring the transversal and on-axis distributions of the detected light power.

Mention as well that for \( \alpha_{rs} z >> 1 \), according to Eqs. (7) and (9a-b), \( J(\tilde{r}, z) \propto z^{-3} \) when \( \alpha_{rs} z << \gamma^2 \), \( J(\tilde{r}, z) \propto z^{-4} \) when \( \chi z >> \gamma^2 \), and \( w^2(z) \propto z^3 \).

When the values of \( \alpha \), \( \chi \) and \( \alpha \) have already been obtained from the experimental data, the determination of \( g \) and \( \alpha_{rs} \) is straightforward. So, in the case of Gaussian indicatrix \( \chi_G = \alpha_{rs} \) we have

\[
\alpha_s = \alpha - \alpha, \quad g = g_G = 1 - \alpha_{rs} / \alpha_s.
\]  

(11a)

In the case of Henyey-Greenstein indicatrix:

\[
\alpha_s = \alpha - \alpha, \quad g = g_{HG} = (1 - 3 \chi_{HG} / \alpha_s)^{1/2}, \quad \alpha_{rs} = \alpha_s (1 - g).
\]  

(11b)

The relations (11a) and (11b) show that in the latter case the value of the \( g \)-factor is smaller, and that of \( \alpha_{rs} \) is larger than in the former case.

At equal values of \( g \) the inequality \( \chi_{HG} < \chi_G \) is obviously in power. Then, when \( \alpha_{rs} z >> 1 \), according to Eqs. (7), (9a), and (9b) the detected-power distribution \( J(\tilde{r}, z) \) for a Henyey-Greenstein indicatrix is narrower, with respectively higher on-axis light intensity, than the one for a Gaussian indicatrix. Such an effect has also been obtained earlier by simulations in Ref. 7. Note as well that the results obtained experimentally and by statistical modeling in the same work 7 outline a behavior of \( J(\tilde{r}, z) \sim z^{-4} \) for \( \alpha_{rs} z > 1 \).

### 3. EXPERIMENTAL SET-UP AND MEASUREMENTS PERFORMED

The experimental set-up is represented in Fig. 1. It makes use as light source of a laser diode emitting a nearly collimated optical beam of about 1mm radius and 27mW light power, and wavelength of 850nm. The tissue-like turbid media of different optical properties are prepared by dilution of different amounts of 20% Intralipid (IL) emulsion by water. Then a 12cm x 12cm x 22cm plexiglass box is filled by any of the mixtures obtained. The axis of the incident laser beam is perpendicular to the “frontal” 12cm x 12cm wall of the container. The forward propagating light power inside the container is measured by using an optical fiber of 0.1mm core diameter introduced into the turbid medium. The fiber is connected with an optical radiometer (Laser precision corp., USA) with a RqP-546 silicon probe in external locking.

![Experimental Set-up](image)

Figure 1. Photograph of the experimental set-up, sketch of a collimated normally-incident laser beam diffusing in a semi-infinite turbid medium (a), and block-scheme of the experimental set-up (b).
regime, 14bits ADC and a computer for appropriate data processing. By a transversal radial scan of the fiber at each stepwise-varied depth of interest we obtained a set of data (distribution of the received light power) for any of the prepared turbid media. The scanning procedure in three mutually perpendicular (x, y, and z) directions is implemented by using three long travel stages Thorlabs LTS 300/M ensuring a minimum sampling step of 4μm. As it is mentioned above, the cross-sectional intensity distribution of the laser beam and the receiving directional diagram of the fiber in air have also been measured and shown to have approximately Gaussian shape (Fig. 2). The angle of view of the fiber in air is estimated to be $\gamma \sim 12^\circ$ i.e., 0.21rad. Then its numerical aperture $NA=\sin \gamma \sim 21$. If the diluted Intralipid emulsion is considered as watery medium with refractive index $n \sim 1.34$, the angle of view of the fiber in these media will be $\gamma = \arcsin(NA/n) \sim 9^\circ$, i.e., 0.16 rad.

The errors in the determination of the parameters of the light beam and, correspondingly, of the turbid medium may be conditioned by different factors. The external locking regime ensures a negligibly small fluctuation-due statistical error of measuring the light power. The error due to external and stray light background is also negligible. The measurement of the light power at relatively large depths, say $>14$cm shows that the background power is orders of magnitude lower than that concerning the optical signals of interest measured at smaller depths. Moreover, the measured background power is taken into account in the

Figure 2. Transversal two-dimensional (a) and radial (b) intensity distributions of the laser beam and directional receiving diagram (c) of the optical fiber measured experimentally and fitted by Gaussian curves.
calculation of $\alpha_{rs}$. Possible mispositioning of $\pm 2 \, \mu m$, according to manufacturer’s information, during the scan of the points of measurement would not lead as well to noticeable errors.

The recorded cross-sectional radial distributions of the light beam intensity are symmetric in general. Nevertheless, some slight misalignment is possible of the scanning directions leading to error of $\pm 1{\text{–}}2 \, \mu m$ in the determination of the beam width. This in turn would lead to an error of about $10\%$ in the determination of $\alpha_{rs}$.

The imperfections of the receiver directional diagram may also be a source of errors.

One more source of errors at relatively small measurement depths (e.g., $<1{\text{–}}2 \, cm$) are the inhomogeneities in the transversal radial distribution of the laser beam intensity. At relatively large depths (e.g., $>7 \, cm$) the influence of the side walls of the box will be sensible leading to additional change of the transversal intensity distribution and width of the light beam. Therefore, the estimation of $\alpha_{rs}$ we have performed is based on data obtained at depths between 2-3cm and 6-7cm. Then, an individual error is not excluded at all.

In any case, a high-accuracy determination of the optical characteristics of a turbid medium would require at least the use of well-defined Gaussian laser beam and receiver directional diagram, well-adjusted scanning system, and large-size plexiglass container.

4. ANALYSIS OF THE EXPERIMENTAL DATA

The turbid media under consideration here are prepared by dilution of different amounts (from 3ml to 120ml) of 20\% IL emulsion in 2300ml water. The wavelength of the laser radiation concerned is $\lambda=850nm$. For IL content from 3ml to 60ml, the in-depth profile of the on-axis detected power exhibits a well-distinguishable initial nearly exponential-falloff region (Fig. 3a) whose extent decreases with the increase of the IL concentration. For the concentrations above 30ml per 2300ml water the exponential-falloff extent is already below 1cm (Fig. 3b). The corresponding detected-power transversal distributions (Fig. 3c) consist of two components, the decaying unscattered light beam on a pedestal of scattered light. A similar picture has been observed in Ref. 6. The exponential-falloff region allows one to determine the attenuation coefficient $\alpha_{l}$ by using Eq. (10b). The estimated here values of $\alpha_{l}$, depending on the IL concentration, are represented graphically in Fig. 4a. As one may intuitively expect, for these relatively low concentrations (from 3ml to 30ml IL content) the dependence obtained is linear. Note that similar low-concentration behavior of $\alpha_{l}$ is observed in Ref.7.

With increasing the depth in the dilution, after passing a transient area, we go into the region of strong scattering, where the on-axis detected light power falls as $z^{-4}$ rather than $z^{-3}$ (Figs. 3e and 3f). At these depths the light beam already consists of practically entirely scattered light. The lateral detected-power distribution has already a bell-shaped, like Gaussian form (see also in 6) whose $e^{-1}$ half-width $w$ increases with $z$ nearly as $z^{3/2}$ (Figs. 3c and 3d). The experimental determination of $w$ allows one to determine the parameter $\chi$ on the basis of the first relation (10a). This in turn allows one on the basis of Eqs. (11a) and (11b), at known $\alpha_{g}$ (see above) and under the assumption that $\alpha_{t}$ is negligibly small, to determine the values of $\alpha_{rs}$ and $g$ for both Gaussian and Henyey-Greenstein indicatrices. The corresponding estimates of $\alpha_{rs}=\alpha_{rsG}$ and $\alpha_{rs}=\alpha_{rsHG}$ obtained here for IL amounts from 9 ml to 60ml are represented graphically in Fig. 4b. As it is seen there, the values of $\alpha_{rs}$ go through a maximum at about 30 ml IL content. Similar nonlinear behavior, with increasing the IL concentration, has also been noticed in Ref. 7, where it is interpreted, e.g., as due to aggregation of the IL scatterers. A little more detailed comment on such phenomena is given for instance in Ref. 8, Sec. 3.5.2. Such a supposition is in accordance with the fact that the values obtained for $g_{G}$ and $g_{HG}$ pass minimum at about 15ml IL content as it is shown in Fig. 4c.

Since the value of $\alpha_{t}$ is orders of magnitude less than $\alpha_{l}$ and $\alpha_{rs}$, it is probably comparable with and even smaller than the measurement errors. Its estimation on the basis of the second relation (10a) is possible perhaps when using well-shaped Gaussian laser beam and directional diagram of the fiber as well as larger-size plexiglass box.
Figure 3. Features of the light beam diffusion in turbid dilutions of 9 ml and 30 ml of 20% Intralipid emulsion in 2300 ml water: (a) and (b), in-depth profiles of the on-axis detected light power $J(0, z)$; (c) and (d), cross-sectional radial profiles $J(0, y, z)$ of the detected light power, at different depths $z$ in the dilution; (e) and (f), illustrations of the dependencies $J(0, z) \propto z^{-3}$ or $z^{-4}$. 
Figure 4. Experimentally estimated values of the extinction $\alpha_t$ (a) and reduced-scattering $\alpha_{rs}$ (b) coefficients and of the g-factor (c) of the investigated turbid media versus diluted Intralipid amount in ml.

5. CONCLUSION

The results obtained in the work show that in general the small-angle approximation could adequately describe the laser beam propagation through the considered turbid media. The analytical expressions obtained for the lateral and on-axis distributions of the detected forward-propagating light power allow one in principle to estimate, on the basis of the experimental data, the values of $\alpha_t$, $\alpha_{rs}$, $g$ and even $\alpha_a$. It is shown that at relatively low turbidity the values of $\alpha_t$ depend linearly on the diluted IL amount. At relatively high turbidity the values of $\alpha_{rs}$ change nonlinearly with the IL amount, passing through maximum around 30ml. Such a behavior correlates with that of $g_G$ and $g_{HG}$ having minimum around 15ml IL content. It is also shown analytically, in agreement with the results from simulations and experiments performed formerly, that at equal $g$ factors, in the case of Henyey-Greenstein indicatrix the propagating trough a medium light beam is narrower, with higher on-axis light intensity compared to the case of Gaussian indicatrix. When the light beam is the same, in the former case the value of $g$ is smaller and that of $\alpha_{rs}$ is larger in comparison with the latter case. A further improvement of the estimation accuracy is achievable when using well-defined Gaussian laser beam and receiving directional diagram of the fiber. One should also investigate the influence of the vessel sizes on the experimental results. Thus, conditions may be found near those of a semi-infinite turbid medium.

The investigations performed in the work are important for the development of methods for measuring the optical characteristics of turbid media such as tissues and experimental tissue-like phantoms. They would also be especially useful in the process of establishing the laws governing the radiative transfer inside the optically investigated biological objects.
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