Statistical modeling of deconvolution procedures for improving the resolution of measuring electron temperature profiles in tokamak plasmas by Thomson scattering lidar

Tanja N. Dreischuh*, Ljuan L. Gurdev, Dimitar V. Stoyanov
Institute of Electronics, Bulgarian Academy of Sciences, 72 Tzarigradsko shosse, Sofia, Bulgaria

ABSTRACT

The potentialities are investigated, by statistical modeling, of deconvolution techniques for high-resolution restoration of electron temperature profiles in fusion plasma reactors like Joint European Torus (JET) measured by Thomson scattering lidar using the center-of-mass wavelength approach. The sensing laser pulse shape and the receiving-system response function are assumed to be exponentially-shaped. The plasma light background influence is taken into account as well as the Poisson fluctuations of the photoelectron number after the photocathode enhanced in the process of cascade multiplying in the employed microchannel photomultiplier tube. It is shown that the Fourier-deconvolution of the measured long-pulse (lidar-response-convolved) lidar profiles, at relatively high and low signal-to-noise ratios, ensures a higher accuracy of recovering the electron temperature profiles with three times higher range resolution compared to the case without deconvolution. The final resolution scale is determined by the width of the window of an optimum monotone sharp-cutoff digital noise-suppressing (noise-controlling) filter applied to the measured lidar profiles.

Keywords: Thomson scattering lidar, fusion plasma diagnostics, electron temperature profiles

1. INTRODUCTION

The electron temperature $T_e$ and density $n_e$ distributions in the torus are basic characteristics of the tokamak fusion plasma. They are conditioned by the modes of heating and confinement of the high-temperature plasma as well as by the different oscillatory movements of the plasma particles sometimes leading to the appearance of crucial instabilities. Thus, the $T_e$ and $n_e$ profiles are not only important factors of the development and the efficiency of the fusion process but indicators as well of the dynamic plasma state. So far, the most appropriate approach to their simultaneous express determination in a remote contactless way is the Thomson scattering (TS) lidar approach. It allows one to obtain the $T_e$ and $n_e$ profiles along a line of sight through the torus core. The minimum range resolution interval achievable by the contemporary core TS lidars is about 12-15 cm. Such a resolution is relatively good in general, but is insufficient for resolving small-scale inhomogeneities and the edge pedestal areas of $T_e$ and $n_e$ profiles in the so-called high-confinement mode (H-mode) of operation of the tokamak reactors.

A way of improving the range resolution of the TS lidars is based on the use of deconvolution techniques for recovering the high-resolution lidar profiles. The deconvolution procedures, however, increase the influence of the noise. Therefore, to achieve acceptable recovered profiles one should apply a final filtering that lowers the sensing resolution to some compromise extent.

The main purpose of the present work is to outline by statistical modeling some optimal conditions under which the deconvolution techniques lead to satisfactory high-resolution restoration of the $T_e$ profiles measured by the center-of-mass wavelength (CMW) method. The sensing laser pulse shape and the receiving-system response function are assumed to be exponentially-shaped. The plasma light background influence is taken into account as well as the Poisson fluctuations of the photoelectron number after the photocathode enhanced in the process of cascade multiplying in the employed microchannel photomultiplier tube.

2. THEORETICAL BACKGROUND

The determination of the electron temperature and density profiles in fusion plasma along a lidar line of sight (LOS) is

*tanjad@ie.bas.bg; phone +359 2 9795867; fax +359 2 9753201; www.ie-bas.dir.bg
based on the analysis of the information provided by the Thomson scattering lidar profiles (the time-to-range resolved profiles of the received backscattered light power). For this purpose, the single-scattering lidar equation which describes the relation between the measured lidar profile, the parameters of the lidar system, and the characteristics of the investigated high-temperature plasma along the LOS should be solved. It is usually accepted that the range resolution of the lidars is of the order of the spatial size of the effective lidar pulse response, which is a convolution of the sensing laser pulse shape and the pulse response function of the receiving electronics. So, in order to achieve a better resolution, shorter sensing pulses and faster registration electronics should be used. When the effective pulse response shape of the lidar is shorter than the least longitudinal variation scale of the characteristics of the fusion plasma the short-pulse lidar equation is in power. In the opposite case, the lidar equation describing the lidar return has the form of the convolution of the short-pulse lidar profile with the system response shape. The necessity of extremely short pulses may be avoided using deconvolution techniques for retrieving the short-pulse lidar profile on the basis of the profiles obtained with relatively longer effective system pulses.

### 2.1 Short-pulse lidar equation

The received by lidar power of the backscattered radiation of wavelength $\lambda_i$, at the moment $t=t(R)$ after the pulse emission, could be written in the form $P[\lambda_i,t(R)]=\hbar\nu_i$, where $N=N[\lambda_i,t(R)]$ is the photoelectron detection rate, $h$ is the Planck’s constant, $\nu_i=c/\lambda_i$, and $c$ is the speed of light; $t(R)$ is an unambiguous linear function of the LOS coordinate $R=R(t)$ of the corresponding scattering volume. Then, the short-pulse lidar equation describing the photoelectron detection rate for the spectral interval $[\lambda_i,\lambda_i+d\lambda_i]$ has the form:

$$N[\lambda_i,t(R)]d\lambda_i = \frac{N_0c}{2}K_n(\lambda_i,\lambda_s)\Delta\Omega(R)\eta(\lambda_s, R)\beta[\lambda_i,\lambda_s, n_e(R), T_e(R)]\frac{\Delta\lambda_i}{\lambda_i}d\lambda_i,$$

where $N_0$ is the number of photons in the sensing laser pulse, $K_n(\lambda_i,\lambda_s)=K(\lambda_i)K(\lambda_s)K(\lambda_s)EQE(\lambda_s)$, $K(\lambda_i), K(\lambda_s), K(\lambda_s)$ and $EQE(\lambda_s)$ are respectively the wavelength-dependent optical transmittance of the plasma-irradiating path, the optical transmittance of the scattered-light collecting path, the receiver filter spectral characteristic, and the effective quantum efficiency of the photon detection, $\Delta\Omega(R)$ is the solid angle of collection of the backscattered radiation as a function of the distance along the line of sight, $\eta(\lambda_i,R)$ is the lidar receiving efficiency, $n_e(R)$ is the electron concentration profile along the LOS, $\lambda_i$ is the wavelength of the sensing laser radiation, $\beta[\lambda_i,\lambda_s, n_e(R), T_e(R)]$ is the Thomson backscattering coefficient (at an angle $\pi$) normalized by $\lambda_s$, at a distance $R$ and a wavelength $\lambda_s$. The analytical expression for the Thomson backscattering spectrum for high-temperature relativistic plasma is given by:

$$\beta[\lambda_i,\lambda_s, n_e(R), T_e(R)] = \frac{n_e(R)n_0^2}{\sqrt{\pi}\lambda_i}v_{th}(R)\left(1+\frac{15}{16}\frac{v_{th}^2(R)}{c^2}+\frac{105}{512}\frac{v_{th}^4(R)}{c^4}\right)^{-1}\frac{(\lambda_i/\lambda_s)^4}{(1+\lambda_i/\lambda_s)}\times\exp\left[-\frac{c^2}{v_{th}^2(R)}\left((\lambda_i/\lambda_s)^{1/2}+(\lambda_s/\lambda_i)^{1/2}-2\right)^2q[\lambda_i,\lambda_s, T_e(R)]\right],$$

where $n_0$ is the classical electron radius, $v_{th}(R)=[2k_BT_e(R)/m_e]^{1/2}$ is the mean thermal velocity of the electrons, $T_e(R)$ is the electron temperature profile along the LOS, $m_e$ is the electron rest mass, $k_B$ is the Boltzmann constant, and $q[\lambda_i,\lambda_s, T_e(R)]$ is the depolarization term accounting for the relativistic depolarization effects on the backscattered radiation. For scattering at $180^\circ$ the depolarization can be expressed in terms of exponential integral $E_n(p)$:

$$q[\lambda_i,\lambda_s, T_e(R)]=1+2pe^p[E_3(p)−3E_5(p)]=1+\frac{1}{2}\left[\frac{p^3}{2}−\frac{p^2}{2}−p−1\right]+p^2\left(1−\frac{p^2}{4}\right)e^pE_1(p),$$

$$p = \frac{m_e^2}{2k_BT_e(R)}\left(\sqrt{\lambda_s/\lambda_i} + \sqrt{\lambda_i/\lambda_s}\right), \quad \text{and} \quad E_n(p) = \int_1^\infty e^{-px^n}dx.$$

If we consider $M$ receiving spectral channels with wavelength intervals $[\lambda_{1m},\lambda_{2m}]$ ($m=1,...,M$), the mean return signal in each of them, in number of photoelectrons per second, can be written as
\[ N_m[\lambda_s, R(t)] = \frac{N_0 c}{2} \Delta \Omega[R(t)] \int_{\lambda_s}^{\lambda_s+2\lambda_s} K_n(\lambda_y, \lambda_s) \eta(\lambda_s, R(t)) \beta[\lambda_y, \lambda_s, n_c[R(t)], T_c[R(t)]] \frac{\lambda_y}{\lambda_s} d\lambda_s. \quad (4) \]

### 2.2. Long-pulse lidar equation

Usually, the laser pulse duration and the detector response time are of the order of several hundred picoseconds and should be taken into account. So, at arbitrarily long effective pulse-response shape of the lidar with respect to the characteristic spatial variation scales in plasma, the lidar equation describing the long-pulse photoelectron rate \( N[\lambda_s, R(t)] \) has the form:

\[
N[\lambda_s, R(t)] = \left( \frac{2}{c} \right) \int_{0}^{R(t)} s(t - 2R'/c) N(\lambda_s, R') dR',
\]

where

\[
s(t-t') = \int_{0}^{t'} g(t-t'') f(t'') d\tau'' \quad (6)
\]

is an effective pulse response (an effective pulse shape) of the lidar system as a whole which is a convolution of the sensing laser pulse shape \( f(\theta) \) and the pulse response function of the receiving electronics \( g(\theta) \). The laser pulse shape \( f(\theta) \) is defined as the pulse power shape \( F_p(\theta) \) normalized to the pulse energy \( E_p = \int_{0}^{\infty} F_p(\theta) d\theta \). Obviously,

\[
\int_{0}^{\infty} f(\theta) d\theta = 1.
\]

The pulse response of the receiving electronics \( g(\theta) \) is defined in an analogous way,

\[
\int_{0}^{\infty} g(\theta) d\theta = 1.
\]

The effective durations of \( f(\theta) \) and \( g(\theta) \) are defined respectively as

\[
T_f = \int_{0}^{\infty} f(\theta) d\theta / f_m \quad \text{and} \quad T_d = \int_{0}^{\infty} g(\theta) d\theta / g_m,
\]

where \( f_m \) and \( g_m \) are corresponding shape maxima. The effective pulse response duration \( T_s = \int_{0}^{\infty} s(\theta) d\theta / s_m \) determines the spatial sensing resolution of the Thomson scattering lidar system; \( s_m \) is the shape maximum.

For a spectral interval \([\lambda_s, \lambda_s+2\lambda_s]\), on the basis of Eqs.(4) and (5) we obtain the following expression of the detected signal photoelectron rate

\[
N_m[\lambda_s, \lambda_s+2\lambda_s, R(t)] = \left( \frac{2}{c} \right) \int_{0}^{R(t)} \left[ 2(R-R') / c \right] N_m(\lambda_s, \lambda_s+2\lambda_s, R') dR'
\]

\[
= N_0 \int_{0}^{R(t)} dR' \left[ 2(R-R') / c \right] \Delta \Omega[R'] \int_{\lambda_s}^{\lambda_s+2\lambda_s} K_n(\lambda_y, \lambda_s) \eta(\lambda_s, R(t)) \beta[\lambda_y, \lambda_s, n_c[R(t)], T_c[R(t)]] \frac{\lambda_y}{\lambda_s} d\lambda_y. \quad (7)
\]

### 2.3. Improving the long-pulse sensing resolution by deconvolution techniques

As already mentioned, the use of long sensing pulses and receiving electronics with pulse responses whose length exceeds the least variation scale of the plasma inhomogeneities leads to a loss of information about the temperature profile structure. The range resolution of the TS lidar systems could be improved by using lasers emitting shorter pulses and faster photodetectors and digitizers. In order to obtain the resolution interval of the order of 3-5 cm, the system response function should have an effective duration \( \sim 200-330 \) ps. The hardware improvement of the TS lidar system may be avoided using deconvolution techniques for retrieving the short-pulse lidar profiles on the basis of the ones obtained with relatively longer effective system response. They consist in solving Eqs.(5) or (7) with respect to the short-pulse profile \( N[\lambda_s, R(t)] \) or \( N_m[\lambda_s, \lambda_s+2\lambda_s, R(t)] \) at measured long-pulse profile \( N[\lambda_s, R(t)] \) or \( N_m[\lambda_s, \lambda_s+2\lambda_s, R(t)] \) and known (measured or estimated) system response \( s(t) \). Since the sampling interval \( \Delta t \) of the analog-to-digital converters (ADCs) is assumed to be small compared to the system response time \( \tau_s \), we would achieve an improved lidar resolution scale of less than \( c \tau_s / 2 \). At a sufficiently low noise level, the resolution scale achieved would be even of the order of \( c \Delta t / 2 \). The determination of \( N[\lambda_s, R(t)] \) or \( N_m[\lambda_s, \lambda_s+2\lambda_s, R(t)] \) allows one to use the methods for processing and interpretation of the TS lidar data, including the CMW approach, described below.
Different deconvolution techniques could be used for improving the resolution of lidars when the sampling intervals are shorter than the system response pulse. The Fourier-transformation technique is a traditional approach to solving the integral equations (5) or (7). As a result, the following Fourier deconvolution algorithm is obtained for retrieval, e.g., of $N_m[\lambda_{s1m}, \lambda_{s2m}, R(t)]$:

$$N_m[\lambda_{s1m}, \lambda_{s2m}, R(t)] = \int_{-\infty}^{\infty} \left[ \tilde{N}_{\text{im}}(\omega)/\tilde{s}(\omega) \right] \exp(-j\omega t) d\omega,$$

where $\tilde{s}(\omega) = \int_{-\infty}^{\infty} s(t') \exp(j\omega t') dt'$ and $\tilde{N}_{\text{im}}(\omega) = \int_{-\infty}^{\infty} N_{\text{im}}(t') \exp(j\omega t') dt'$ are Fourier transforms of $s(t)$ and $N_{\text{im}}(t)$, respectively; $j$ is imaginary unit. The numerical procedure of calculating $N_m[\lambda_{s1m}, \lambda_{s2m}, R(t)]$ on the basis of Eq.(8) includes the fast-Fourier-transformation algorithm for $s(t)$ and $N_{\text{im}}(t)$ sampled by ADC interval $\Delta t_c$.

The feasibilities of this algorithm in the case of lidar remote sensing of the atmosphere are investigated theoretically and by computer simulations, as well as by processing of real lidar data in the works. A central, expected result of the investigations performed there is that the noise influence on the recovered lidar profiles may be essential, especially when the noise spectrum is wider than the spectrum of the system response shape. Then, as it is shown in the cited papers, a preliminary low-pass filtering of the received (convolved) lidar profile improves significantly the results from the deconvolution. Certainly, the width of the window of the filter employed should be shorter than the system response length in order to ensure a higher accuracy of recovering the short-pulse lidar profiles and respectively of the $T_e$ profiles.

### 2.4. Plasma light background

The Thomson scattering lidar signal is accompanied by the plasma light background that is serious source of error in the determination of $T_e$. Its emissivity spectrum per unit solid angle, mainly due to the bremsstrahlung, is given by the expression:

$$\frac{dE}{d\Omega} = \frac{0.95 \times 10^{-19}}{\lambda} n_e^2(R) Z_{\text{eff}}(R) [k_B T_e(R)]^{-1/2} \exp\left(-\frac{hc}{\lambda k_B T_e(R)}\right) \tilde{g}_{g\beta}(\lambda, T_e),$$

where $Z_{\text{eff}}(R)$ is the effective ion charge, the quantities $k_B T_e$ and $hc/\lambda$ are in eV, $\exp[-hc/(\lambda k_B T_e)] \approx 1$ and $\tilde{g}_{g\beta}(\lambda, T_e)$ is the so-called Gaunt factor that depends weakly on $T_e$ and on the radiation wavelength $\lambda$, and accounts for the quantum effects, the electron screening of nuclei, etc. For the photoelectron rate characterizing the parasitic background due to plasma light penetrating into the $m$-th spectral channel we obtain the following expression:

$$N_{\text{im}}(\lambda_{s1m}, \lambda_{s2m}) = 6.25 \times 10^{-21} A_D \Delta \Omega_D \int \frac{dR n_e^2(R) [k_B T_e(R)]^{-1/2}}{R} \left( \int_{\lambda_{s1m}}^{\lambda_{s2m}} d\lambda K_{\beta}(\lambda) K_{\beta}(\lambda) EQE(\lambda) \lambda^{2/3} \ln \left[ k_B T_e(R)/(13.6h^2 c^2 / \lambda^2) \right] \right)^{1/3},$$

where $A_D$ is the photon detector effective area and $\Delta \Omega_D$ is the solid angle determined by the relative aperture of the receiving optics.

In order to take into account additional background light sources, an enhancement factor is included in the simulations.

### 2.5. Determination of the electron temperature $T_e$ on the basis of center-of-mass wavelength approach

Let us consider $M$ spectral intervals $[\lambda_{s1m}, \lambda_{s2m}]$ ($m=1,...,M$) from the relativistic Thomson backscattering spectrum from fusion plasma, which are selected by the receiving optical system of the lidar. The central wavelength of the $m$-th interval is $\lambda_m = (\lambda_{s1m} + \lambda_{s2m})/2$. Then the center-of-mass wavelength (CMW) $\lambda_{\text{CM}}$ defined as:

$$\lambda_{\text{CM}}(T_e) = \left( \sum_m \lambda_m N_m / \sum_m N_m \right)$$

is unambiguous function of the electron temperature (see also Fig.3 below). An explicit writing of Eq.(11) is
\[
\lambda_{CM}(T_e) = \frac{\sum_m \lambda_{z_m} \int K_n(\lambda_i, \lambda_s) \eta(\lambda_s, R(t)) \beta[\lambda_i, \lambda_s, n_e[R(t)], T_e[R(t)]] \frac{\lambda_s}{\lambda_i} d\lambda_s}{\sum_m \lambda_{z_m}}.
\]  

The linear error propagation approach\(^5\) leads to the following expression of the rms error \(\delta T_e\) in the determination of \(T_e\) on the basis of the dependence \(\lambda_{CM} = f(T_e)\):

\[
\delta T_e = \left[ \frac{d \ln \lambda_{CM}(T_e) / d T_e}{\sqrt{\sum_m N_{jm} T_d}} \right]^{-1/2} \left\{ \sum_m \left( \frac{\lambda_{z_m} - \lambda_{CM}}{\lambda_{CM}} \right)^2 N_{jm} T_d (1 + N_{bm} / N_{fm}) \right\}^{1/2},
\]

where

\[
N_{jm}[\lambda_{z1m}, \lambda_{z2m}, R(t)] = (2/c) \int_0^{R(t)} f(2(R - R')/c) N_m(\lambda_{z1m}, \lambda_{z2m}, R') dR'
\]

is the convolution of the laser pulse shape and the short-pulse signal photoelectron rate. The determinant temporal factor in Eq.(13) is \(T_d\) because it is in practice the signal integration time interval\(^5\). In case of applying deconvolution techniques for recovering the short-pulse lidar profiles and thus for obtaining more accurate \(T_e\) profiles, in Eq.(13) \(T_d\) should be replaced by the width \(\tau_e > T_d\) of the time-domain window of the final noise-controlling filter, and the convolved photoelectron rate \(N_{jm}\) should be replaced by the short-pulse one, \(N_m\). In addition, the background photoelectron rate \(N_{bm}(R)\) in Eq.(13) should be multiplied by a factor \(\Phi(T_e/\tau_e) > 1\) that is an increasing function of the ratio \(T_e/\tau_e\). Such a noise enhancement is due to the fact that the background is initially smoothed (integrated) only by the receiving electronics response function while the deconvolution is performed using total lidar response function including the laser pulse shape.

An estimate of the signal-to-noise ratio (SNR) for the \(m\)-th spectral channel could be written as follows\(^5\):

\[
SNR_m = \left\{ N_{jm} T_d / [1 + N_{bm} / N_{fm}] \right\}^{1/2}
\]

in the case of convolved lidar profiles, and

\[
SNR_m = \left\{ N_m \tau_f / [1 + N_{bm} \Phi(T_f/\tau_f) / N_m] \right\}^{1/2}
\]

in the case of deconvolved lidar profiles.

### 3. SIMULATIONS

The characteristic parameters of the plasma and the TS lidar used in the simulations are chosen to be close to those of the core TS lidar system on the Joint European Torus (JET)\(^1\).\(^2\). The sensing laser pulse-beam is assumed to have wavelength \(\lambda_i = 694\) nm and energy \(E_0 = N_0 h c / \lambda_i = 1\) J, and to be injected horizontally along the plasma midplane. The minor radius of the torus, along the LOS, is supposed to be 1 m, i.e., the plasma is supposed to occupy the region between \(R = 2\) m and \(R = 4\) m (\(R\) being the radial distance from the center of the torus)\(^1\).\(^5\). The number of receiving spectrometer channels is chosen to be six. Their absolute spectral responses, including the effective quantum efficiency (EQE) of the detectors, are also close to those of JET TS core lidar\(^1\).\(^3\). In particular, the detectors considered in the simulations are multialkali microchannel plate photomultiplier tubes (MCP-PMTs) with response times of about 650 ps and EQE equal to 0.005 for channel 1 and 0.02 for the other five channels. TS spectrum is observed within the wavelength region from 350 nm to 850 nm. To correct the collection efficiency the values of the solid angle of acceptance given in Ref.3 are used. They vary from 0.005 sr, at \(R = 2\) m, to 0.007 sr at \(R = 4\) m. The irradiating and collecting paths optical transmittances assumed are \(K_0(\lambda_i) = 0.75\) and \(K_0(\lambda_o) = 0.25\), respectively. The detector’s etendue \(E = A_\delta \Delta \Omega_\phi\) needed for the estimation of the plasma bremsstrahlung photoelectron rate is assumed to have a value of \(\sim 0.32\) cm\(^2\)sr. The factor of reducing the plasma bremsstrahlung conditioned by the plasma torus observation pupil is supposed to be 0.3. The effective atomic number of an equivalent plasma ion is chosen to be \(Z_{eq} = 2\). The bremsstrahlung background is added multiplied by an enhancement factor of 2 in order to take into account additional background light sources.
In the simulations below the temporal sampling interval $\Delta t_s$ is supposed to be 200 ps (3 cm spatial interval). In general, it can have different values, according to the analog-to-digital converters used in the experiment. The main purpose of the simulations is to show the improvement of the accuracy of recovering the temperature and density profiles achieved after the application of the deconvolution algorithms in comparison with the accuracy of the profiles obtained using the convolved lidar profiles.

A pair of models of the temperature and density profiles used in the simulations is presented in Fig.1. Both models consist of a smooth component — parabola with parameters chosen to simulate the real plasma conditions. Additionally, the $T_e(R)$ profile has a multiscale high-resolution component superimposed on the smooth component in order to illustrate the improvement of the resolution using the deconvolution and the relation between the resolution and noise level. The central electron density is varied in the range $n_e = 2 \div 9 \times 10^{19}$ m$^{-3}$ to simulate different plasma conditions (see Fig.1b).

The sensing laser pulse is chosen to have the following shape normalized to the pulse energy $-f(\theta) = (\theta/\tau_1)^2 \exp(-\theta/\tau_1)$ for $\theta \geq 0$ and $f(\theta) = 0$ for $\theta < 0$, where $\tau_1$ is a time constant. The exponentially-shaped pulse considered can be a good approximation of various real asymmetric laser pulses (see, e.g., Refs.16-17). The same model is also used for the shape of the receiving electronics response $g(\theta) = (\theta/\tau_2)^2 \exp(-\theta/\tau_2)$ for $\theta \geq 0$, and $g(\theta) = 0$ for $\theta < 0$, where $\tau_2$ is another time constant. The Fourier spectrum modulus of the above pulse shapes is equal to $(1+\omega^2 \tau_2^2)^{-1}$, i.e., it has no zeros, which is favorable for applying Fourier-deconvolution algorithm. The effective durations of $f(\theta)$ and $g(\theta)$ in this case are $T_l = \int_0^\infty f(\theta) d\theta / f_m = e \tau_1$ and $T_d = \int_0^\infty g(\theta) d\theta / g_m = e \tau_4$, respectively. The values of the time constants $\tau_1$ and $\tau_4$ are chosen so that $T_l$ and $T_d$ to be respectively about 350 ps ($\tau_1 = 130$ ps) and 810 ps ($\tau_4 = 300$ ps). Then the effective duration of the resulting system response shape $T_s$ will be about 1 ns which corresponds to 15 cm range resolution cell of the TS lidar. The models of the laser pulse shape, the receiving electronics response shape and the TS lidar system response shape are shown in Fig.2.

In order to apply the CMW approach to determine the plasma temperature profiles, the reference function $\lambda_{CM}(T_e)$ should be known. It is determined on the basis of the temperature dependence of the TS spectrum and is presented in Fig.3 for temperatures up to 10 keV. In the case of long-pulse sensing, when the pulse length exceeds the spatial scale of the temperature inhomogeneities, the temperature information provided by the lidar profiles from the different spectral channels will be distorted. Correspondingly, the recovered temperature profiles will also be distorted with respect to the true ones. The role of the deconvolution here is to reduce, as much as possible at the corresponding noise level, the convolution-due distortions of the recovered $T_e$ profiles.

The Monte-Carlo simulations are performed in the following way. First, the mean values $N_n(R)$ of the TS signal in each spectral channel are determined and then convolved with the laser pulse shape in order to account for the real pulse duration. Next, the mean background photoelectron count rate $N_n(R)$ is evaluated. Then, assuming Poisson statistics of the signal and background photoelectron counts within a $\Delta t_e$-long interval and using random-number generator, a number

![Fig.1. Models of the electron temperature profile (a) and of two electron density profiles (b) as functions of the radial distance from the center of the torus.](image-url)
of $J$ realizations of the TS signal $N_{fm}(R) \Delta t_s$ and background $N_{bm}(R) \Delta t_s$ photoelectron counts is produced. Further, the receiving electronics response function is taken into account performing the convolution with it of the background and count rates in each channel. At last, the obtained realizations of the lidar profiles including the background are deconvolved with the system response function in order to improve the range resolution. The center of mass wavelength as a function of the radius along the LOS is determined according to Eq.(11) on the basis of the deconvolved profiles, and is used together with the reference function $\lambda_{CM}(T_e)$ for obtaining $J$ estimates $\hat{T}_e(R)$ of the electron temperature profile $T_e(R)$, $j=1,\ldots,J$. Then, an estimate $\delta \hat{T}_e(R)$ of the measurement error is obtainable as

$$\delta \hat{T}_e(R) = \left\{ J^{-1} \sum_{j=1}^{J} [\hat{T}_e(R) - T_e(R)]^2 \right\}^{1/2} .$$

It should be noted that in order to simulate correctly the detection of the analog signals, the convolved profiles are calculated almost ideally by a computing step much less than $\Delta t_s$. After that the real ADC step $\Delta t_s$ is taken into account when performing the discretization of the convolved signals. The mean short-pulse and the registered long-pulse lidar profiles for the six spectral channels are shown in Figs.4a,b. The plasma background noise is taken also into account and is added to the TS signals. In Fig.4c the same profiles for the 4th spectral channel are presented along with the signal which would be registered with a $\Delta t_s$ - wide response function of the receiving electronics ("measured" short-pulse lidar profile) and the deconvolved lidar signal.

**Fig.4.** TS lidar profiles: (a) mean short-pulse lidar profiles, (b) measured long-pulse lidar profiles, (c) statistical models of mean short-pulse, and "measured" short-pulse and long-pulse lidar profiles for the 4th spectral channel; $n_e = 9 \times 10^{19} \text{ m}^{-3}$.
As an illustration of the deconvolution effect, the \( T_e \) profiles restored in the absence of noise using the convolved and deconvolved lidar profiles are shown in Fig.5. As it is seen, in the case of long pulses the direct use of the recorded lidar profiles leads to lowering of the range resolution and to significant distortions in the restored electron temperature profiles. After applying the deconvolution to the lidar profiles, the \( T_e \) profiles could be determined with an improved resolution scale, which can be even of the order of the sampling interval \( c\Delta t/2 \) of the analog-to-digital converters at a sufficiently low noise level.

![Fig.5. Electron temperature profiles restored in absence of noise on the basis of the convolved (a) and deconvolved (b) lidar profiles.](image)

In practice, the registered Poisson-fluctuating lidar return consists of the TS return signal and plasma light background, accompanied with the detector noise (see Fig.4). Therefore, some type of background subtraction and noise filtering is necessary to ensure a satisfactory quality of the restored profiles. However, the filtering procedure lowers the range resolution. The resolution cell will be already of the order of the width \( W \) of the window of the filter employed. To retain a satisfactory range resolution the value of \( W \) should be less than the least variation scale (along the line of sight) of the temperature profile. Then the restored temperature and density profiles are minimally distorted with respect to the true ones. Different low-pass digital filters are used in the numerical simulations. Results presented below are obtained using filters with \( 2\Delta t_s \) and \( 3\Delta t_s \) wide windows for smoothing the recorded lidar profiles.

In the next Fig.6 the profiles of the electron temperature restored on the basis of the convolved and deconvolved lidar profiles for one realization of the Poisson noise are presented. It is well seen that the restored temperature profile on the basis of convolved lidar profiles (Fig.6a) is essentially distorted with respect to the original model. At the same time, the temperature profile restored on the basis of deconvolved lidar profiles (Fig.6b) is disguised by strongly increased fluctuations. In order to suppress the deconvolution-due increase of the noise, noise controlling filters have been applied (Figs.6c,d) ensuring acceptable accuracy and resolution of the restored electron temperature profiles. It is seen in Fig.6d that even \( 2\Delta t_s \) wide filter window (corresponding to 6 cm range resolution) ensures good quality of the obtained \( T_e \) profile. The theoretical errors presented in these figures are estimated without the factor \( \varphi (T_e/\tau_f) \) to be taken into account for deconvolved profiles. When using convolved profiles the theoretically and numerically estimated \( T_e \) restoration errors are not shifted with respect to each other because then the factor \( \varphi (T_e/\tau_f) \) is not of importance. The same is the behavior of the restored profiles in the case of lower electron concentration \( n_e = 2 \times 10^{19} \text{ m}^{-3} \) (see Fig.7). Because of the lower signal-to-noise ratio in this case, the quality of the restored profiles is somewhat lower compared to the former case (\( n_e = 9 \times 10^{19} \text{ m}^{-3} \)).

The results of applying the deconvolution approach in the case of increased sensing pulse energy (\( E_0 = 3 \text{ J} \)) are shown in Fig.8, where it is seen that the restoration accuracy is higher due to the higher signal-to-noise ratio. The plasma inhomogeneities with a variation scale of the order of 6 cm (Fig.8c) could be reliably detected on the basis of correctly restored temperature profile structure.

As it was already discussed, the range resolution of the lidar can be successfully improved with a suitable upgrade of the laser source, the detectors, and the data acquisition system. In the next Fig.9 we present the results from the simulations performed assuming the same effective lidar pulse response (the same sensing laser pulse shape and pulse response function of the receiving electronics) as above but shorter sampling step \( \Delta t_s = 40 \text{ ps} \). So, the initial lidar resolution is again...
about 15 cm. The restored electron temperature profile on the basis of the convolved lidar profiles is given in Fig.9a. As seen, the distortion of the obtained $T_e$ profile with respect to the real one is similar to this presented above. In this case, however, applying the deconvolution without any filtering of the lidar profiles leads to much higher noise fluctuations because of the smaller discretization step. So, in order to suppress this increase of the noise, it is necessary to apply digital filters with windows of the order of 7-10 $\Delta t$. Fig.9b shows the results when a smooth monotone filter with $9\Delta t$-wide window is applied. This corresponds to a 5.4 cm range resolution interval which is again similar to those as in the case of ADC with 200 ps sampling step.

The results from these simulations show that the upgrade of the data acquisition modules alone in the TS lidar system would not lead to an essential improvement of the achievable range resolution even with a deconvolution. The most important factor is the signal-to-noise ratio. So, if there is no possibility to upgrade the laser source and detectors to shorten the lidar response function, one should search for other ways for improving the signal-to-noise ratio – e.g. by increasing the sensing pulse energy - in order to increase the accuracy of measuring the core plasma electron temperature and density profiles.

**Fig.6.** Electron temperature profiles restored on the basis of the convolved (a) and deconvolved lidar profiles without filtering (b), and on the basis of the deconvolved lidar profiles smoothed by a monotone sharp-cutoff digital filter (with $W=3c\Delta t/2$) (c) and by a moving average filter (with $W=2c\Delta t/2$) (d); inset – theoretically estimated relative rms errors compared to the numerically obtained ones; $n_e = 9\times10^{19}$ m$^{-3}$.
4. CONCLUSION

The investigations performed in this work show that Fourier-deconvolution procedures combined with appropriate low-pass filtering, applied to the measured Thomson scattering lidar profiles lead to several (2-3) times better resolution of recovering electron temperature profiles in fusion plasma, under conditions of plasma light background and amplification-enhanced Poisson noise. The convolution-due systematic errors are essentially corrected for and an
acceptable restoration accuracy is achieved allowing one to reveal characteristic inhomogeneities in the distribution of the electron temperature within the plasma torus. It is also shown that, naturally, because of higher signal-to-noise ratio (stronger lidar return) the deconvolution accuracy increases with the increase of the electron concentration and the sensing pulse energy. This means that the deconvolution approach would be especially appropriate for processing data from a new generation of fusion reactors, such as ITER and DEMO, characterized by considerably higher electron concentration and sensing pulse energy compared to these achievable in JET.

ACKNOWLEDGEMENTS

This work, supported by the European Communities under the Contract of Association between EURATOM and INRNE (Bulgaria), was carried out within the framework of the European Fusion Development Agreement. The views and opinions expressed herein do not necessarily reflect those of the European Commission. The authors would also like to acknowledge the useful discussions with Marc Beurskens and Mike Walsh.

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